

# On new physics in $\Delta\Gamma_d$

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**ABSTRACT:** Motivated by the recent measurement of the dimuon asymmetry by the DØ collaboration, which could be interpreted as an enhanced decay rate difference in the neutral  $B_d$ -meson system, we investigate the possible size of new-physics contributions to  $\Delta\Gamma_d$ . In particular, we perform model-independent studies of non-standard effects associated to the dimension-six current-current operators  $(\bar{d}p)(\bar{p}'b)$  with  $p, p' = u, c$  as well as  $(\bar{d}b)(\bar{\tau}\tau)$ . In both cases we find that for certain flavour or Lorentz structures of the operators sizable deviations of  $\Delta\Gamma_d$  away from the Standard Model expectation cannot be excluded in a model-independent fashion.

**KEYWORDS:** Mostly Weak Interactions: B-Physics, CP violation, Rare Decays, Beyond Standard Model

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## 1 Introduction

The experimental measurements of the like-sign dimuon asymmetry by the DØ collaboration in 2010 and 2011 [1–3] triggered much interest in the flavour-physics community. If the value of the measured asymmetry  $A_{\text{CP}}$  is interpreted solely as a CP-violating effect in mixing of neutral  $B_{d,s}$  mesons ( $a_{sl}^{d,s}$ ),

$$A_{\text{CP}} \propto A_{sl}^b \equiv C_d a_{sl}^d + C_s a_{sl}^s, \quad (1.1)$$

then using  $C_d \simeq 0.59$  and  $C_s \simeq 0.41$  the experimental number from 2011 [3] deviates by  $3.9\sigma$  from the latest Standard Model (SM) prediction [4]. One finds, however, that new-physics contributions in  $\Delta B = 2$  transitions alone cannot explain the large central value of the like-sign dimuon asymmetry, since such a large enhancement would violate model-independent bounds (see e.g. [5, 6]).

In [7] the interpretation of the like-sign dimuon asymmetry within the SM was revisited. It was found that (1.1) should be modified to take into account previously neglected

contributions proportional to the decay rate differences  $\Delta\Gamma_d$  and  $\Delta\Gamma_s$ . These arise from interference of  $B$ -meson decays with and without mixing. The modified result reads

$$A_{\text{CP}} \propto A_{sl}^b + C_{\Gamma_d} \frac{\Delta\Gamma_d}{\Gamma_d} + C_{\Gamma_s} \frac{\Delta\Gamma_s}{\Gamma_s}. \quad (1.2)$$

Numerical values for the coefficients  $C_{\Gamma_d}$  and  $C_{\Gamma_s}$  can be extracted from [7] and [8]. The sign of  $C_{\Gamma_d}$  is such that a positive value of  $\Delta\Gamma_d$  gives a negative contribution to  $A_{\text{CP}}$  and  $C_{\Gamma_s}$  turns out to be negligible. It follows that the measured like-sign dimuon asymmetry is not simply proportional to the semi-leptonic asymmetries  $a_{sl}^{d,s}$ , as assumed in [1–3].

Very recently the DØ collaboration presented a new measurement [8] of the coefficients  $C_d$ ,  $C_s$  and  $C_{\Gamma_d}$  ( $C_{\Gamma_s}$  was neglected) and more importantly of the inclusive single-muon charge asymmetry  $a_{\text{CP}}$  and the asymmetry  $A_{\text{CP}}$ . The result is

$$a_{\text{CP}} = (-0.032 \pm 0.042 \pm 0.061)\%, \quad A_{\text{CP}} = (-0.235 \pm 0.064 \pm 0.055)\%. \quad (1.3)$$

If one uses (1.2) as a starting point and assumes the SM value for  $\Delta\Gamma_d/\Gamma_d$ , then the new measurement can be used to extract the following result for CP violation in mixing

$$A_{sl}^b = (-0.496 \pm 0.153 \pm 0.072)\%, \quad (1.4)$$

which differs by  $2.8\sigma$  from the SM prediction [4]. The result in (1.4) is considerably smaller than the value  $A_{sl}^b = (-0.787 \pm 0.172 \pm 0.093)\%$  presented in 2011 [3]. The reason for the noticeable shift of the central value in  $A_{sl}^b$  is that in [3] (as well as in all other previous experimental and theoretical analyses) the contribution proportional to  $\Delta\Gamma_d$  in (1.2) was neglected.

Stronger statements can be obtained from the data in [8], if different regions for the muon impact parameter (denoted by the index  $i$ ) are investigated separately instead of averaging over them, as done to get the values in (1.3). It is then possible to extract individual values for  $a_{sl}^d$ ,  $a_{sl}^s$  and  $\Delta\Gamma_d$  from the measurements of the  $a_{\text{CP}}^i$  and  $A_{\text{CP}}^i$ . One finds [8]

$$a_{sl}^d = (-0.62 \pm 0.43)\%, \quad a_{sl}^s = (-0.82 \pm 0.99)\%, \quad \frac{\Delta\Gamma_d}{\Gamma_d} = (0.50 \pm 1.38)\%. \quad (1.5)$$

The result differs from the combined SM expectation for the three observables by  $3.0\sigma$ . If one instead assumes that the semi-leptonic asymmetries  $a_{sl}^d$  and  $a_{sl}^s$  are given by their SM values, then the decay rate difference  $\Delta\Gamma_d$  measured by [8] using (1.2) is

$$\frac{\Delta\Gamma_d}{\Gamma_d} = (2.63 \pm 0.66)\%, \quad (1.6)$$

which differs by  $3.3\sigma$  from the SM prediction.

All in all, the new DØ measurements still differ from SM expectations at the level of  $3\sigma$  and part of this tension could be related to an anomalous enhancement of the decay rate difference  $\Delta\Gamma_d$ . This raises the question to what extent  $\Delta\Gamma_d$  can be enhanced by beyond the SM effects, without violating other experimental constraints. In fact, this is an interesting question in its own right. While potential new-physics contributions to the related quantity

$\Delta\Gamma_s$  have been studied in detail (see e.g. [9–16]) and turned out to be strongly constrained by different flavour observables — at most enhancements of 35% are allowed experimentally — possible new-physics effects in  $\Delta\Gamma_d$  have received much less attention. An exception is the article [17] which emphasises that a precision measurement of  $\Delta\Gamma_d$  would provide an interesting window to new physics.

The main goal of this paper is to close the aforementioned gap by performing model-independent studies of two types of new-physics contributions to  $\Delta\Gamma_d$  that could in principle be large. As a first possibility we consider new-physics contributions due to current-current operators  $(\bar{d}p)(\bar{p}'b)$  with  $p, p' = u, c$ , allowing for flavour-dependent and complex Wilson coefficients. We study carefully the experimental constraints on each of the coefficients that arise from hadronic two-body decays such as  $B \rightarrow \pi\pi, \rho\pi, \rho\rho, D^*\pi$ , the inclusive  $B \rightarrow X_d\gamma$  decay and the dimension-eight contributions to  $\sin(2\beta)$  [18] as extracted from  $B \rightarrow J/\psi K_S$ . Our analysis shows that large deviations in  $\Delta\Gamma_d$  and  $a_{sl}^d$  are currently not ruled out, if they are associated to the current-current operator involving two charm quarks. We emphasise that our general model-independent framework covers the case of violations of the unitarity of the Cabibbo-Kobayashi-Maskawa (CKM) matrix. As a second possibility we analyse the constraints on new-physics contributions from operators of the form  $(\bar{d}b)(\bar{\tau}\tau)$ . We show that since the existing constraints imposed by tree-level and loop-level mediated  $B$ -meson decays such as  $B \rightarrow \tau^+\tau^-$  and  $B^+ \rightarrow \pi^+\mu^+\mu^-$  are quite loose, sizable modifications of  $\Delta\Gamma_d$  and  $a_{sl}^d$  are possible also in this case, in particular if they arise from vector operators. From a purely phenomenological point of view it thus seems much easier to postulate absorptive new physics in the  $B_d$ -meson system rather than in the  $B_s$ -meson system.

Our paper is organised as follows. In Section 2 we briefly set our notation and collect the experimental values and SM predictions for the mixing quantities. In Section 3 we illustrate the principle differences between  $\Delta\Gamma_d$  and  $\Delta\Gamma_s$ . We turn to the aforementioned model-independent new-physics studies in Sections 4 and 5, while Section 6 contains our conclusions. In Appendix A we give the SM result for  $\Delta\Gamma_d$ , including a detailed breakdown of theoretical uncertainties. The input values employed in our numerical calculations are summarised in Appendix B.

## 2 Mixing formalism

Mixing phenomena in the  $B_q$ -meson system, with  $q = d, s$ , are related to the off-diagonal elements of the complex mass matrix  $M_{12}^q$  and decay rate matrix  $\Gamma_{12}^q$ . We choose the three physical mixing observables as the mass difference  $\Delta M_q$ , the width difference  $\Delta\Gamma_q$  and the flavour specific (or semi-leptonic) CP asymmetries  $a_{sl}^q$ . The general expressions of these observables are

$$\Delta M_q = 2 |M_{12}^q|, \quad \Delta\Gamma_q = 2 |\Gamma_{12}^q| \cos \phi_q, \quad a_{sl}^q = \left| \frac{\Gamma_{12}^q}{M_{12}^q} \right| \sin \phi_q, \quad (2.1)$$

with the mixing phase  $\phi_q = \arg(-M_{12}^q/\Gamma_{12}^q)$ . The above equations are valid up to corrections of  $\mathcal{O}(1/8 |\Gamma_{12}^q/M_{12}^q|^2 \sin^2 \phi_q)$ , which is smaller than  $10^{-7}$  in the SM for the  $B_q$ -meson systems [4].

Quantity		$q = d$		$q = s$	
$\Delta M_q$ [ps <sup>-1</sup> ]	SM	$0.543 \pm 0.091$	[4]	$17.3 \pm 2.6$	[4]
	exp.	$0.507 \pm 0.004$	[21] HFAG	$17.69 \pm 0.08$	[21] HFAG
$\Delta\Gamma_q/\Gamma_q$ [%] [%] [%]	SM	$0.42 \pm 0.08$	[4]	—	
	exp.	$1.5 \pm 1.8$	[21] HFAG	—	
		$0.5 \pm 1.38$	[8] DØ	—	
		$-4.4 \pm 2.7$	[22] LHCb	—	
$\Delta\Gamma_q$ [ps <sup>-1</sup> ]	SM	$0.0029 \pm 0.0007$	see (A.2)	$0.087 \pm 0.021$	[4]
	exp.	$0.0059 \pm 0.0079$	[8, 21]	$0.081 \pm 0.011$	[21] HFAG
$a_{sl}^q$ [10 <sup>-4</sup> ]	SM	$-4.1 \pm 0.6$	[4]	$1.9 \pm 0.3$	[4]
	exp.	$6 \pm 17_{-23}^{+38}$	[23] BaBar	$-6 \pm 50 \pm 36$	[26] LHCb
		$68 \pm 45 \pm 14$	[24] DØ	$-112 \pm 74 \pm 17$	[25] DØ
$\phi_q$	SM	$-0.085 \pm 0.025$	[4]	$0.0042 \pm 0.0013$	[4]
	exp.	—		$[-1.04, +0.96]^*)$	[26] LHCb
		—		$[-1.34, -0.58]^*)$	[25] DØ
$\beta, -2\beta_s$	SM	$0.446_{-0.037}^{+0.016}$	[27]	$-0.03676_{-0.00144}^{+0.00128}$	[27]
	exp.	$0.374 \pm 0.014$	[21] HFAG	$0.04_{-0.13}^{+0.10}$	[21] HFAG
		—		$0.01 \pm 0.07 \pm 0.01$	[28] LHCb

**Table 1.** Collection of SM predictions and measurements (exp.) of  $B_q$ -meson mixing observables for  $q = d, s$ . \*)As obtained from the combination of measurements of  $\Delta M_s$ ,  $\Delta\Gamma_s$  and  $a_{sl}^s$  in the table. See text for more details.

To study new-physics effects it is convenient to parameterise the general expressions for the mixing matrices in such a way that the SM contributions are factored out. We employ the notation introduced in [19, 20] and write

$$\begin{aligned}
M_{12}^q &= M_{12}^{q,\text{SM}} \Delta_q, & \Delta_q &= |\Delta_q| e^{i\phi_q^\Delta}, \\
\Gamma_{12}^q &= \Gamma_{12}^{q,\text{SM}} \tilde{\Delta}_q, & \tilde{\Delta}_q &= |\tilde{\Delta}_q| e^{-i\phi_q^{\tilde{\Delta}}}.
\end{aligned}
\tag{2.2}$$

The observables are then modified with respect to their SM predictions according to

$$\frac{\Delta M_q}{\Delta M_q^{\text{SM}}} = |\Delta_q|, \quad \frac{\Delta\Gamma_q}{\Delta\Gamma_q^{\text{SM}}} = |\tilde{\Delta}_q| \frac{\cos \phi_q}{\cos \phi_q^{\text{SM}}}, \quad \frac{a_{sl}^q}{a_{sl}^{q,\text{SM}}} = \frac{|\tilde{\Delta}_q|}{|\Delta_q|} \frac{\sin \phi_q}{\sin \phi_q^{\text{SM}}}, \tag{2.3}$$

where the mixing phase is given by

$$\phi_q = \phi_q^{\text{SM}} + \phi_q^\Delta + \phi_q^{\tilde{\Delta}}. \tag{2.4}$$

We now compare the SM results with the experimental measurements, contrasting the situations in the  $B_d$ -meson and  $B_s$ -meson systems. For this purpose we have collected the

SM results of the mixing observables and the corresponding experimental measurements in Table 1. Concerning  $\Delta M_q$ , the central values agree very nicely, but the experimental errors are much smaller than the theoretical ones, which are dominated by hadronic uncertainties. The experimental numbers are the current world averages obtained by the HFAG collaboration [21] and incorporate various measurements performed at experiments from ALEPH to LHCb that are all consistent with each other.

In the case of the width difference, the SM prediction of  $\Delta\Gamma_s$  agrees quite well with experimental world average [21] that combines measurements from LHCb [28], ATLAS [29], CDF [30] and DØ [31]. The very good agreement between theory and experiment shows that the Heavy Quark Expansion (HQE) used to calculate  $\Delta\Gamma_s$  works to an accuracy of  $\mathcal{O}(25\%)$  or better. The SM prediction for  $\Delta\Gamma_d$  itself was not explicitly given in [4] but can be extracted from the numerical code used in that work. For this reason, we give a short description of the components that go into it and also a detailed analysis of different sources of uncertainties in Appendix A. While the theory treatment of  $\Delta\Gamma_d$  is in complete analogy to  $\Delta\Gamma_s$ , experimental measurements are much more challenging due to its small size, which follows from the hierarchy of the CKM matrix elements involved. The bounds on  $\Delta\Gamma_d$  given in Table 1 stem from the latest HFAG average and two more recent investigations from DØ [8] and LHCb [22]. The measurements of  $\Delta\Gamma_d$  are still subject to large errors and significant deviations from the SM calculation are not yet ruled out. We return to this in a moment.

The experimental situation for the semi-leptonic CP asymmetries is more complicated, because the current HFAG values  $a_{sl}^d = (7 \pm 27) \cdot 10^{-4}$  and  $a_{sl}^s = (-171 \pm 55) \cdot 10^{-4}$  [21] are based on the traditional interpretation (1.1) of the dimuon asymmetry through CP violation in mixing alone, whereas the correct interpretation seems to be the one given in (1.2). We therefore quote in Table 1 separately the direct measurements in semi-leptonic decays from BaBar [23], DØ [24, 25] and LHCb [26]. These results for semi-leptonic CP asymmetries are now complemented by values based on the interpretation (1.2) of the dimuon asymmetry given in (1.5). Obviously, the experimental measurements of the semi-leptonic asymmetries do little to restrict possible new-physics contributions.

Alternatively, one might also compare the SM predictions for the mixing phases with experimental constraints via the relation  $\phi_q = \tan^{-1}(a_{sl}^q \Delta M_q / \Delta\Gamma_q)$ . Because of the large uncertainty in  $\Delta\Gamma_d$ , the experimental numbers do not provide any stringent constraint on  $\phi_d$  at present.<sup>1</sup> For the phase  $\phi_s$  the 68% confidence level (CL) ranges are given in Table 1.

In addition to the three mixing observables  $\Delta M_q$ ,  $\Delta\Gamma_q$  and  $a_{sl}^q$ , the phase of  $M_{12}^q$  affects time-dependent CP asymmetries of neutral  $B_q$ -meson decays. Neglecting CP violation in mixing and corrections of  $\mathcal{O}(|\Gamma_{12}^q/M_{12}^q|^2)$ , the interference between mixing and decay,

$$\frac{\text{Br}(\bar{B}_q(t) \rightarrow f) - \text{Br}(B_q(t) \rightarrow f)}{\text{Br}(\bar{B}_q(t) \rightarrow f) + \text{Br}(B_q(t) \rightarrow f)} = S_f \sin(\Delta M_q t) - C_f \cos(\Delta M_q t), \quad (2.5)$$

gives rise to the direct and the mixing-induced CP asymmetries  $C_f$  and  $S_f$ , respectively.

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<sup>1</sup>If one takes the 68% CL range of the DØ value for the semi-leptonic asymmetry, then small negative values of  $\phi_d$  are excluded.

Within these approximations, especially  $S_f$  is sensitive to the new physics phase  $\phi_q^\Delta$  of  $M_{12}^q$

$$S_f = \frac{2 \operatorname{Im} \left[ e^{-i(2 \arg(V_{tq}^* V_{tb}) + \phi_q^\Delta)} \rho_f \right]}{1 + |\rho_f|^2}, \quad (2.6)$$

where  $\rho_f \equiv \bar{\mathcal{A}}_f / \mathcal{A}_f$  and  $\bar{\mathcal{A}}_f$  ( $\mathcal{A}_f$ ) is the hadronic amplitude of the  $\bar{B}_q$  ( $B_q$ )-meson decay. In cases where the hadronic amplitude is dominated by a single weak phase, such as in the SM,  $S_f$  can be cleanly related to the corresponding CKM parameters. This is for instance the case for decays which are dominated by the tree  $b \rightarrow c\bar{c}s$  transition or the gluonic penguin  $b \rightarrow q\bar{q}s$  ( $q = u, d, s$ ) transitions.

The two well-known examples of  $b \rightarrow c\bar{c}s$  mediated tree decays are  $B_d \rightarrow J/\psi K_S$  and  $B_s \rightarrow J/\psi \phi$ , both of which measure the relative phase between the hadronic decay amplitude and  $M_{12}^q$ . In this case one has

$$S_f = -\sin \left( \pm 2\beta_q^f \right) = -\sin \left( \pm 2\beta_q + \delta_q^\Delta + \delta_q^{\text{peng, NP}} + \delta_q^{b \rightarrow c\bar{c}s} \right), \quad (2.7)$$

with  $+$  for  $q = d$  and  $-$  for  $q = s$ . The dominant SM amplitude contributes  $\beta_d = \beta = \arg(-V_{cb}^* V_{cd} / V_{tb}^* V_{td})$  and  $\beta_s = \arg(-V_{tb}^* V_{ts} / V_{cb}^* V_{cs})$ , respectively. New-physics effects can change either the phase  $\phi_q^\Delta$  of  $M_{12}^q$  (see (2.2)), whose contribution is denoted by  $\delta_q^\Delta$ , the phase of  $\mathcal{A}_f$  or both. The new-physics contributions to the hadronic decay amplitude can be further subdivided into the ones from penguin contributions  $\delta_q^{\text{peng, NP}}$  and the new-physics phase  $\delta_q^{b \rightarrow c\bar{c}s}$  appearing in the  $b \rightarrow c\bar{c}s$  tree-level amplitude. The SM penguin contributions are expected to be quite small (see e.g. [32]). This general parametrisation implies that the  $\delta_q^i$  are actually dependent on both weak and strong phases specific to the final state  $f$ . In the current work we are only interested in new physics that does not introduce visible effects in the penguin sector and furthermore consider only interactions that affect the  $b \rightarrow c\bar{c}d$  tree-level decay, but not  $b \rightarrow c\bar{c}s$ . In consequence, we set  $\delta_q^{\text{peng, NP}}$  and  $\delta_q^{b \rightarrow c\bar{c}s}$  to zero in (2.7).

The lesson to learn from the above comparisons is that with the exception of  $\Delta M_d$ , the mixing observables in the  $B_d$ -meson system are rather loosely constrained experimentally. There is thus in principle plenty of room for beyond the SM contributions to  $\Delta\Gamma_d$  and  $a_{sl}^d$ , which depend besides  $M_{12}^d$  also strongly on the non-standard effects in  $\Gamma_{12}^d$ . Within the SM,  $\Gamma_{12}^d$  is determined by  $\Delta B = 1$  operators of dimension six, predominantly those arising from tree-level  $W$ -boson exchange  $b \rightarrow f_1 \bar{f}_2 d$  with  $f_i = u, c$ . Beyond the SM, other contributions are conceivable, however the masses of  $f_i$  should allow for the formation of intermediate on-shell states in order to affect  $\Gamma_{12}^d$ . Below we will study constraints on such absorptive contributions within a model-independent, effective field theory framework for the two cases  $f_i = u, c$  and  $f_1 = f_2 = \tau$ .

A further important point to keep in mind is that  $\Gamma_{12}^d$  is related to a product of two  $\Delta B = 1$  operators, while  $M_{12}^d$  is determined from  $\Delta B = 2$  operators that are obtained by integrating out high-virtuality particles. In the effective field theory approach the two types of operators are independent of one another. Motivated by this observation we will study the case where new physics manifests itself to first approximation only in terms of  $\Delta B = 1$  interactions of the form  $b \rightarrow f_1 \bar{f}_2 d$ , which change  $\Gamma_{12}^d$  (i.e.  $\tilde{\Delta}_d \neq 1$ ), while the leading

dimension-six  $\Delta B = 2$  operators are assumed to remain SM-like. In this way we can get an idea of how large the maximum effects in  $\Gamma_{12}^d$  can be. Of course, in explicit new-physics models the Wilson coefficients of both types of operators will generically be modified, which can lead to correlated effects in  $M_{12}^d$  and  $\Gamma_{12}^d$ , thereby reducing the possible shifts in  $\Gamma_{12}^d$ . Studying such correlations requires to specify a concrete model, which goes beyond the scope of this work. However, even in our approach, the dimension-six  $\Delta B = 1$  operators give rise to dimension-eight  $\Delta B = 2$  operators due to operator mixing, and consequently, induce also new-physics contributions to  $M_{12}^d$  (i.e.  $\Delta_q \neq 1$ ). We will study these effects in Section 4 and show that they are phenomenologically harmless for the effective interactions considered in our analysis.

### 3 Comparison of $\Delta\Gamma_d$ and $\Delta\Gamma_s$

The first important observation is that  $\Delta\Gamma_d$  is triggered by the CKM-suppressed decay  $b \rightarrow c\bar{c}d$ , whose inclusive branching ratios reads  $(1.31 \pm 0.07)\%$  (based on the numerical evaluation in [33]), while  $\Delta\Gamma_s$  receives the dominant contribution from the CKM-favoured decay  $b \rightarrow c\bar{c}s$ , which has an inclusive branching ratio of  $(23.7 \pm 1.3)\%$  [33]. This means that a relative modification of  $\Gamma(b \rightarrow c\bar{c}d)$  by 100% shifts the total  $b$ -quark decay rate  $\Gamma_{\text{tot}}$ <sup>2</sup> by around 1% only, while a 100% variation in  $\Gamma(b \rightarrow c\bar{c}s)$  results in an effect of roughly 25% in the same observable. Large enhancement of the  $b \rightarrow c\bar{c}d$  decay rate can therefore be hidden in the hadronic uncertainties of  $\Gamma_{\text{tot}}$ , while this is not possible in the case of the  $b \rightarrow c\bar{c}s$  decay rate.

Second, the CKM structure of  $\Gamma_{12}^d$  and  $\Gamma_{12}^s$  are notably different within the SM. Separating  $\Gamma_{12}^q$  into the individual contributions with only internal charm quarks ( $\Gamma_{12}^{cc,q}$ ) or up quarks ( $\Gamma_{12}^{uu,q}$ ) and one internal charm quark and one up quark ( $\Gamma_{12}^{cu,q}$ ), one can write the SM contribution to  $\Gamma_{12}^q$  as follows

$$\Gamma_{12}^{q,\text{SM}} = - \left[ (\lambda_c^q)^2 \Gamma_{12}^{cc,q} + 2 \lambda_c^q \lambda_u^q \Gamma_{12}^{cu,q} + (\lambda_u^q)^2 \Gamma_{12}^{uu,q} \right], \quad (3.1)$$

with  $\lambda_p^q \equiv V_{pq}^* V_{pb}$  and [4]

$$\Gamma_{12}^{cc,d} = 18.85 \text{ ps}^{-1}, \quad \Gamma_{12}^{cu,d} = 20.72 \text{ ps}^{-1}, \quad \Gamma_{12}^{uu,d} = 22.52 \text{ ps}^{-1}. \quad (3.2)$$

The results for the  $\Gamma_{12}^{pp',s}$  coefficients relevant in the  $B_s$ -meson system are obtained from the latter numbers by a simple rescaling with the factor  $(f_{B_s} M_{B_s})^2 / (f_{B_d} M_{B_d})^2 = 1.48$ . Notice that the three values in (3.2) are quite similar, which implies that phase-space effects are not very pronounced.

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<sup>2</sup>We do not distinguish here between the total  $B$ -meson decay rates  $\Gamma_{B_d}$ ,  $\Gamma_{B_s}$  and  $\Gamma_{B^+}$  and the total  $b$ -quark decay rate  $\Gamma_{\text{tot}}$ , because the measured differences are smaller than the current theoretical uncertainties.



Combining the formulae (3.1) and (3.2), we obtain the following numerical expressions

$$\begin{aligned}
\Gamma_{12}^{d,\text{SM}} &= -[(1.60 + 0.00i) + (-0.50 + 1.37i) + (-0.25 - 0.21i)] \cdot 10^{-3} \text{ps}^{-1} \\
&= -[0.85 + 1.16i] \cdot 10^{-3} \text{ps}^{-1}, \\
\Gamma_{12}^{s,\text{SM}} &= -[(42.81 + 0.00i) + (0.72 - 1.97i) + (-0.02 - 0.02i)] \cdot 10^{-3} \text{ps}^{-1} \\
&= -[43.51 - 1.98i] \cdot 10^{-3} \text{ps}^{-1}.
\end{aligned} \tag{3.3}$$

From the first line of the result for  $\Gamma_{12}^{d,\text{SM}}$ , we see that in the  $B_d$ -meson system there is a partial cancellation between the individual contributions, because the two relevant CKM factors are of similar size in this case, i.e.  $|\lambda_c^d| \simeq |\lambda_u^d| = \mathcal{O}(\lambda^3)$  with  $\lambda \simeq 0.22$  denoting the Cabibbo angle. In the case of  $\Gamma_{12}^s$ , on the other hand, the result is fully dominated by the contribution due to  $\Gamma_{12}^{cc,s}$ , since  $|\lambda_c^s| = \mathcal{O}(\lambda^2)$  while  $|\lambda_u^s| = \mathcal{O}(\lambda^4)$ . The observed partial cancellation leads again to the feature that a modification in  $b \rightarrow c\bar{c}d$  will have a much larger effect in  $\Gamma_{12}^d$ , compared to the effect of a similar modification in  $b \rightarrow c\bar{c}s$  in  $\Gamma_{12}^s$ . For instance, a 100% shift in  $\Gamma_{12}^{cc,d}$  leads to an almost 300% modification of  $\Gamma_{12}^d$ , while a 100% change of  $\Gamma_{12}^{cc,s}$  results in a 100% shift of  $\Gamma_{12}^s$  only.

Another way of looking at the mixing systems is to investigate the ratio  $\Gamma_{12}^q/M_{12}^q$ . Using the unitarity of the CKM matrix, i.e.  $\lambda_u^q + \lambda_c^q + \lambda_t^q = 0$ , we find the SM expression

$$\left(\frac{\Gamma_{12}^q}{M_{12}^q}\right)_{\text{SM}} = \left[(-51 \pm 10) + \frac{\lambda_u^q}{\lambda_t^q}(10 \pm 2) + \left(\frac{\lambda_u^q}{\lambda_t^q}\right)^2(0.16 \pm 0.03)\right] \cdot 10^{-4}, \tag{3.4}$$

where the numerical coefficients are identical for the  $B_d$ -meson and  $B_s$ -meson system within errors. The relevant CKM factors are  $\lambda_u^d/\lambda_t^d = -0.033 - 0.439i$  and  $\lambda_u^s/\lambda_t^s = -0.008 + 0.021i$ . It follows that the real part of  $(\Gamma_{12}^q/M_{12}^q)_{\text{SM}}$  and thus  $(\Delta\Gamma_q/\Delta M_q)_{\text{SM}}$  is dominated by the first term in the square bracket of (3.4), which encodes the contribution to  $\Gamma_{12}^{q,\text{SM}}$  involving charm quarks only. The situation is quite different for the imaginary parts of  $(\Gamma_{12}^q/M_{12}^q)_{\text{SM}}$  that arise from the second and third term and determine the size of  $a_{sl}^{q,\text{SM}}$ . The fact that in the SM the semi-leptonic CP asymmetry in the  $B_d$ -meson sector is about 20 times larger than the one in the  $B_s$ -meson system and has opposite sign is hence a simple consequence of  $\text{Im}(\lambda_u^d/\lambda_t^d) \simeq -20 \text{Im}(\lambda_u^s/\lambda_t^s)$ .

The structure of  $\Gamma_{12}^{q,\text{SM}}$  and  $(\Gamma_{12}^q/M_{12}^q)_{\text{SM}}$  also allows one to draw some general conclusions on how new physics can modify  $\Delta\Gamma_{d,s}$  and  $a_{sl}^{d,s}$ . Consider for instance the violation of CKM unitarity  $\lambda_u^q + \lambda_c^q + \lambda_t^q = \Delta_{\text{CKM}}^q$ , a property known from beyond the SM scenarios (see e.g. [34–36]) in which heavy fermions mix with the SM quarks and/or new charged gauge bosons mix with the  $W$  boson.<sup>3</sup> In such models the relation (3.4) would receive a shift that can be approximated by

$$\Delta\left(\frac{\Gamma_{12}^q}{M_{12}^q}\right) \simeq (-51 \pm 10) \left[ \left(1 - \frac{\Delta_{\text{CKM}}^q}{\lambda_t^q}\right)^2 - 1 \right] \cdot 10^{-4}. \tag{3.5}$$

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<sup>3</sup>See also [37] for a recent discussion of a similar point.

Given our imperfect knowledge of some of the elements of the CKM matrix, deviations of the form  $\Delta_{\text{CKM}}^d \simeq \Delta_{\text{CKM}}^s = \mathcal{O}(\lambda^3)$  are not excluded phenomenologically. From (3.5) we then see that such a pattern of CKM unitarity violation can lead to a relative enhancement of  $|\Gamma_{12}^d/M_{12}^d|$  by up to 300%, while in the case of  $|\Gamma_{12}^s/M_{12}^s|$  the relative shifts can be 50% at most. Depending on the phase of  $\Delta_{\text{CKM}}^d/\lambda_t^d$  the new contribution in (3.5) could hence affect  $\Delta\Gamma_d$  and  $a_{sl}^d$  in a significant way, while leaving  $\Delta M_d$ ,  $\Delta M_s$ ,  $\Delta\Gamma_s$  and  $a_{sl}^s$  unchanged within hadronic uncertainties. In fact, in the next two sections we will see that it is possible to find certain effective interactions that support the general arguments presented above.

#### 4 New physics in $\Delta\Gamma_d$ : current-current operators

In the following we derive model-independent bounds on the Wilson coefficients of so-called current-current operators. We write the part of the effective weak Hamiltonian involving these operators as

$$\mathcal{H}_{\text{eff}}^{\text{current}} = \frac{4G_F}{\sqrt{2}} \sum_{p,p'=u,c} V_{pd}^* V_{p'b} \sum_{i=1,2} C_i^{pp'}(\mu) Q_i^{pp'} + \text{h.c.}, \quad (4.1)$$

with  $G_F$  the Fermi constant. The current-current operators are then defined as

$$Q_1^{pp'} = (\bar{d}^\alpha \gamma^\mu P_L p^\beta) (\bar{p}'^{\prime\beta} \gamma_\mu P_L b^\alpha), \quad Q_2^{pp'} = (\bar{d} \gamma^\mu P_L p) (\bar{p}' \gamma_\mu P_L b), \quad (4.2)$$

where  $P_L = (1 - \gamma_5)/2$  projects onto left-handed fields and  $\alpha, \beta$  denote colour indices.

In the SM, the coefficients  $C_i^{pp'}$  are real and depend neither on the quark content  $pp'$  nor on whether the transition is  $b \rightarrow s$  or  $b \rightarrow d$ . On the other hand, a generic new-physics model will give rise to different contributions to each non-leptonic decay channel and one must consider carefully the constraints on the (complex) coefficients  $C_i^{pp'}$  individually. While some ingredients needed for such a study have been mentioned previously in the literature, see for instance [11], it has yet to be carried out in any detail. The goal of this section is to fill this gap by deriving bounds on the  $C_i^{uu}$ ,  $C_i^{uc}$  and  $C_i^{cc}$  coefficients, which multiply the operators governing  $b \rightarrow u\bar{u}d$ ,  $b \rightarrow c\bar{u}d$  and  $b \rightarrow c\bar{c}d$  transitions, respectively.<sup>4</sup> We structure this section by discussing the coefficients  $C_{1,2}^{pp'}$  in turn, examining not only the constraints but also their implications for deviations in  $\Delta\Gamma_d$  from the SM expectation. To do so, we first write

$$C_{1,2}^{ij} = C_{1,2}^{\text{SM}} + \Delta C_{1,2}^{ij}. \quad (4.3)$$

Then, generalising the expressions from Section 3, we obtain

$$\frac{\Gamma_{12}^d}{\Gamma_{12}^{d,\text{SM}}} - 1 = (0.61 - 0.84i) \left[ (\Delta C_2^{cc})^2 + 0.064 \Delta C_2^{cc} \Delta C_1^{cc} + 2.1 \Delta C_2^{cc} - 0.26 \Delta C_1^{cc} + 0.77 (\Delta C_1^{cc})^2 \right]$$

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<sup>4</sup>The coefficient  $C_i^{cu}$  governs  $b \rightarrow u\bar{c}d$  transitions, which are severely CKM suppressed in the SM. Since currently the most stringent bounds on such a coefficient come from  $\Delta\Gamma_d$  itself, we exclude it from the analysis.

$$\begin{aligned}
& - (0.21 - 0.052i) \left[ (\Delta C_2^{uu})^2 + 0.35 \Delta C_1^{uu} \Delta C_2^{uu} + 2.0 \Delta C_2^{uu} \right. \\
& \quad \left. - 0.16 \Delta C_1^{uu} + 1.3 (\Delta C_1^{uu})^2 \right] \\
& + (0.53 + 0.79i) \left[ \Delta C_2^{cu} \Delta C_2^{uc} + 1.05 \Delta C_1^{cu} \Delta C_1^{uc} \right. \\
& \quad + 0.11 (\Delta C_1^{uc} \Delta C_2^{cu} + \Delta C_1^{cu} \Delta C_2^{uc}) \\
& \quad \left. + 1.0 (\Delta C_2^{cu} + \Delta C_2^{uc}) - 0.10 (\Delta C_1^{cu} + \Delta C_1^{uc}) \right]. \quad (4.4)
\end{aligned}$$

Here and in the remainder of the section we use a notation where the Wilson coefficients  $\Delta C_{1,2}^{pp'}$  are to be evaluated at the scale  $m_b = \bar{m}_b(\bar{m}_b) \simeq 4.2 \text{ GeV}$  unless otherwise specified. Given this expression, it is straightforward to calculate the ratio  $\Delta\Gamma_d/\Delta\Gamma_d^{\text{SM}}$  using (2.3).

While it is the coefficients  $\Delta C_{1,2}^{pp'}(m_b)$  which appear in low-energy observables such as (4.4), it is important to keep in mind that these are obtained from the matching coefficients at the new-physics scale  $\Lambda_{\text{NP}}$  through renormalisation-group (RG) evolution. In what follows, we will always present bounds on the coefficients at the scale  $M_W$  for convenience. The leading-logarithmic (LL) evolution connecting the coefficients at the two scales can be written as

$$\begin{aligned}
\Delta C_1^{pp'}(m_b) &= z_+ \Delta C_1^{pp'}(M_W) + z_- \Delta C_2^{pp'}(M_W), \\
\Delta C_2^{pp'}(m_b) &= z_- \Delta C_1^{pp'}(M_W) + z_+ \Delta C_2^{pp'}(M_W),
\end{aligned} \quad (4.5)$$

where

$$z_{\pm} = \frac{1}{2} \left( \eta_5^{\frac{6}{23}} \pm \eta_5^{-\frac{12}{23}} \right), \quad \eta_5 \equiv \frac{\alpha_s(M_W)}{\alpha_s(m_b)}, \quad (4.6)$$

and  $\alpha_s$  should be evaluated in the five-flavour theory. Throughout our work, in deriving the constraints on the individual coefficients from a given observable, we work under the assumption of “single operator dominance” and consider only changes in the coefficients one at a time. E.g. to set constraints on  $\Delta C_1^{uu}(M_W)$  we fix  $\Delta C_2^{uu}(M_W) = 0$ .

The dominant effect of the modifications of the current-current sector considered here is to change  $\Gamma_{12}^d$  from its SM value. However, double insertions of  $\Delta B = 1$  operators also give dimension-eight contributions to  $M_{12}^q$ . It turns out that these are completely negligible numerically with the exception of the contributions from the  $Q_2^{cc}$  operator. We thus postpone a more detailed discussion of these contributions to Section 4.3.

#### 4.1 Bounds on up-up-quark operators

We begin our analysis by deriving constraints on  $C_{1,2}^{uu}$  from  $B \rightarrow \pi\pi$ ,  $\rho\pi$ ,  $\rho\rho$  decays.<sup>5</sup> QCD factorisation provides a tool for calculating various observables in these decays to leading power in  $\Lambda_{\text{QCD}}/m_b$  [39]. The reliability of the factorisation predictions is a subject of

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<sup>5</sup>Constraints on the real and imaginary parts of  $C_{1,2}^{uu}$  can also be derived from studies of the isospin asymmetry in  $B \rightarrow \rho\gamma$  [38]. The obtained bounds would benefit greatly from better measurements of the  $B \rightarrow \rho\gamma$  decay.

debate, and in the following we consider only observables which can be argued to be under theoretical control within this approach.

The process  $B^- \rightarrow \pi^- \pi^0$  is to an excellent approximation a pure tree decay and thus provides strong constraints on the magnitudes and phases of the Wilson coefficients  $C_{1,2}^{uu}$ . A particularly clean probe of tree amplitudes is provided by the ratio [40]

$$R_{\pi^- \pi^0} = \frac{\Gamma(B^- \rightarrow \pi^- \pi^0)}{d\Gamma(\bar{B}^0 \rightarrow \pi^+ \ell^- \bar{\nu}_\ell)/dq^2|_{q^2=0}} \simeq 3\pi^2 f_\pi^2 |V_{ud}|^2 |\alpha_1(\pi\pi) + \alpha_2(\pi\pi)|^2, \quad (4.7)$$

which by construction is free of the uncertainty related to  $|V_{ub}|$  and the  $B \rightarrow \pi$  form factor  $F_0^{B \rightarrow \pi}(0)$ . Here  $\Gamma(B^- \rightarrow \pi^- \pi^0)$  denotes the  $B^- \rightarrow \pi^- \pi^0$  rate,  $d\Gamma(\bar{B}^0 \rightarrow \pi^+ \ell^- \bar{\nu}_\ell)/dq^2|_{q^2=0}$  is the semi-leptonic decay spectrum differential in the dilepton invariant mass  $q^2$ , evaluated at  $q^2 = 0$ . The numerical values of the pion decay constant  $f_\pi$  and of  $|V_{ud}|$  are collected in Table 3.

Ignoring small electroweak penguin amplitudes the ratio  $R_{\pi^- \pi^0}$  measures the magnitude of the sum of the coefficients  $\alpha_{1,2}(\pi\pi)$  of the tree amplitudes [39]. These coefficients are currently known to next-to-next-to-leading order (NNLO) in perturbation theory [41, 42]. Working to NNLO in the SM, but to leading order (LO) in the new-physics contributions  $\Delta C_{1,2}^{uu}$  in (4.3), we obtain the expressions<sup>6</sup>

$$\begin{aligned} \alpha_1(\pi\pi) &= 0.195 - 0.101i + \Delta C_1^{uu} + \frac{\Delta C_2^{uu}}{3}, \\ \alpha_2(\pi\pi) &= 1.013 + 0.027i + \Delta C_2^{uu} + \frac{\Delta C_1^{uu}}{3}. \end{aligned} \quad (4.8)$$

The given SM values correspond to the central values for  $\alpha_{1,2}(\pi\pi)$  presented in [41]. Inserting (4.8) into (4.7), one obtains the approximation

$$R_{\pi^- \pi^0} \simeq (0.70_{-0.08}^{+0.12}) \left[ 1 + 2.20 \operatorname{Re} \Delta C_+^{uu} - 0.13 \operatorname{Im} \Delta C_+^{uu} + 1.21 |\Delta C_+^{uu}|^2 \right] \text{GeV}^2, \quad (4.9)$$

where  $\Delta C_+^{pp'} \equiv \Delta C_1^{pp'} + \Delta C_2^{pp'}$  and we have restored the theoretical error of the SM prediction for  $R_{\pi^- \pi^0}$  as given in [41]. The corresponding experimental value is

$$R_{\pi^- \pi^0} = (0.81 \pm 0.14) \text{GeV}^2, \quad (4.10)$$

which has been derived in [41] based on the information given in [21, 43]. Combining the theoretical expectation (4.9) with the experimental determination allows to directly constrain the magnitude and phase of  $\Delta C_+^{uu}(m_b)$ . In order to turn this constraint into individual bounds on  $\Delta C_{1,2}^{uu}(M_W)$ , we use (4.5) along with the assumption of single operator dominance explained after (4.6). In Figure 1 we show the parameter ranges (blue circular bands) that are allowed at 90% CL using this procedure. We see that  $\mathcal{O}(1)$  effects in  $\Delta C_{1,2}^{uu}(M_W)$  are allowed if they leave the magnitude  $|\alpha_1(\pi\pi) + \alpha_2(\pi\pi)|$  SM-like.

A very effective way to constrain the phases of the Wilson coefficients in addition to their magnitude is to study mixing-induced CP asymmetries  $S_f$  (2.6) in  $B \rightarrow \pi\pi, \pi\rho, \rho\rho$

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<sup>6</sup>Note that  $\alpha_{1,2}$  are interchanged with respect to the common definition in the literature [39], to comply with our choice of operator basis.

transitions. On the other hand, the direct CP asymmetries,  $C_f$ , are suppressed by powers of  $\alpha_s$  and/or  $\Lambda_{\text{QCD}}/m_b$  in QCD factorisation and are difficult to predict quantitatively. The mixing-induced CP asymmetries are directly proportional to  $\sin(2\beta + 2\gamma)$  in the SM, in the limit where the Wilson coefficients of the QCD penguin operators are ignored. When the Wilson coefficients  $\Delta C_{1,2}^{uu}$  are allowed to be complex, this SM relation is altered to include the phase of the coefficients, even at LO in  $\alpha_s$ . The situation is more complicated when penguin corrections are included. However, the penguin-to-tree ratio is of  $\mathcal{O}(30\%)$  in  $B \rightarrow \pi\pi$  and of  $\mathcal{O}(10\%)$  in  $B \rightarrow \rho\pi$  [39], so that the constraints given by these observables can still be considered relatively clean.

We can evaluate the indirect asymmetries in the  $B \rightarrow \pi\pi, \rho\pi$  sectors at NLO in QCD factorisation using the formulae given in [39].<sup>7</sup> We find

$$S_{\pi^+\pi^-} \simeq -(47 \pm 28) \left[ \frac{3.48 \text{Im } r_{\pi^+\pi^-} + 1.87 \text{Re } r_{\pi^+\pi^-}}{1 + 0.87 |r_{\pi^+\pi^-}|^2} \right] \%, \quad (4.11)$$

with

$$r_{\pi^+\pi^-} = \frac{1 + (0.94 - 0.27i) \Delta C_{1/3}^{uu}}{1 + (0.88 + 0.25i) (\Delta C_{1/3}^{uu})^*}, \quad (4.12)$$

and

$$\begin{aligned} S_{\rho\pi} &\equiv \frac{1}{2} (S_{\rho^+\pi^-} + S_{\rho^-\pi^+}) \\ &\simeq (6.9 \pm 34) \left\{ 0.62 \left[ \frac{-27.3 \text{Im } r_{\rho^-\pi^+} + 2.4 \text{Re } r_{\rho^-\pi^+}}{1 + 1.4 |r_{\rho^-\pi^+}|^2} \right] \right. \\ &\quad \left. + 0.38 \left[ \frac{-31.7 \text{Im } r_{\rho^+\pi^-} + 1.7 \text{Re } r_{\rho^+\pi^-}}{1 + 0.7 |r_{\rho^+\pi^-}|^2} \right] \right\} \%, \end{aligned} \quad (4.13)$$

with

$$r_{\rho^-\pi^+} = \frac{1 + (0.99 - 0.10i) \Delta C_{1/3}^{uu}}{1 + (0.96 - 0.12i) (\Delta C_{1/3}^{uu})^*}, \quad r_{\rho^+\pi^-} = \frac{1 + (0.96 + 0.07i) \Delta C_{1/3}^{uu}}{1 + (0.97 + 0.08i) (\Delta C_{1/3}^{uu})^*}. \quad (4.14)$$

We have defined  $\Delta C_{1/3}^{pp'} \equiv \Delta C_2^{pp'} + \Delta C_1^{pp'}/3$  and the central values correspond to the default parameter choice employed in [39], apart from  $\beta$  for which we use  $21.8^\circ$  [27]. The result depends very strongly on the angle  $\gamma$ , for which we use  $\gamma = (70.0 \pm 10)^\circ$ , in line with the current world average [21] obtained from  $B^+ \rightarrow DK^+$  and related processes. The quoted error is derived from the procedure used in [39] and is dominated by the dependence on  $\gamma$ . We have also included in the error the range of values obtained in scenarios S2 to S4 of that work. The current experimental results are [21]

$$S_{\pi^+\pi^-} = -(65 \pm 6)\%, \quad (4.15)$$

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<sup>7</sup>We have also studied the indirect asymmetry  $S_{\rho^+\rho^-}$ . The constraints are qualitatively similar to those from  $S_{\rho\pi}$  and do little to cut down the allowed parameter space, so we exclude them from Figure 1 for simplicity.

and [44]

$$S_{\rho\pi} = (5 \pm 9)\%. \quad (4.16)$$

The 90% CL constraints imposed by  $S_{\pi^+\pi^-}$  are displayed in brown in the panels of Figure 1 and those by  $S_{\rho\pi}$  in red. For the case of  $\Delta C_2^{uu}$  one sees that combining the restrictions from  $R_{\pi^-\pi^0}$  with those from the two indirect CP asymmetries singles out three allowed regions, one of which is the SM-like solution. The restriction on  $\Delta C_1^{uu}$  is much weaker than that on  $\Delta C_2^{uu}$ , which can be understood evaluating (4.5) numerically to find

$$\Delta C_{1/3}^{uu}(m_b) \simeq 1.0 \Delta C_2^{uu}(M_W) + 0.12 \Delta C_1^{uu}(M_W). \quad (4.17)$$

The quantity  $\Delta C_1^{uu}(M_W)$  is multiplied by a small coefficient, so that constraints from quantities such as these indirect CP asymmetries, which only depend on this combination of coefficients, are roughly 10 times stronger for  $\Delta C_2^{uu}$  than for  $\Delta C_1^{uu}$ , when the method of varying only one coefficient at a time is used. While the quantity  $S_{\pi^+\pi^-}$  still cuts out a very small portion of the allowed parameter space for  $\Delta C_1^{uu}$ , it turns out that  $S_{\rho\pi}$  offers no further constraint on  $\Delta C_1^{uu}(M_W)$  and has therefore been omitted from the figure.

Part of the remaining parameter space can be eliminated by the quantity  $R(\rho^-\rho^0/\rho^+\rho^-)$ , which is the ratio of branching ratios of  $B^- \rightarrow \rho^-\rho^0$  and  $\bar{B}^0 \rightarrow \rho^+\rho^-$ . The extension of the QCD factorisation formalism necessary to describe these decays to NLO has been derived in [45, 46]. The results for these decays depend on the parameters  $\alpha_{1,2}(\rho\rho)$ , analogous to (4.8) and as in the  $\pi\pi$  sector these are known to NNLO from [41, 42]. The decays also receive contributions from QCD and electroweak penguin coefficients, which have only been calculated up to the NLO level. Combining all known corrections, one finds

$$R(\rho^-\rho^0/\rho^+\rho^-) = (0.65^{+0.16}_{-0.11}) \left[ \frac{1 + 2.1 \operatorname{Re} \Delta C_+^{uu} + 0.06 \operatorname{Im} \Delta C_+^{uu} + 1.1 |\Delta C_+^{uu}|^2}{1 + 2.0 \operatorname{Re} \Delta C_{1/3}^{uu} + 0.23 \operatorname{Im} \Delta C_{1/3}^{uu} + 1.0 |\Delta C_{1/3}^{uu}|^2} \right]. \quad (4.18)$$

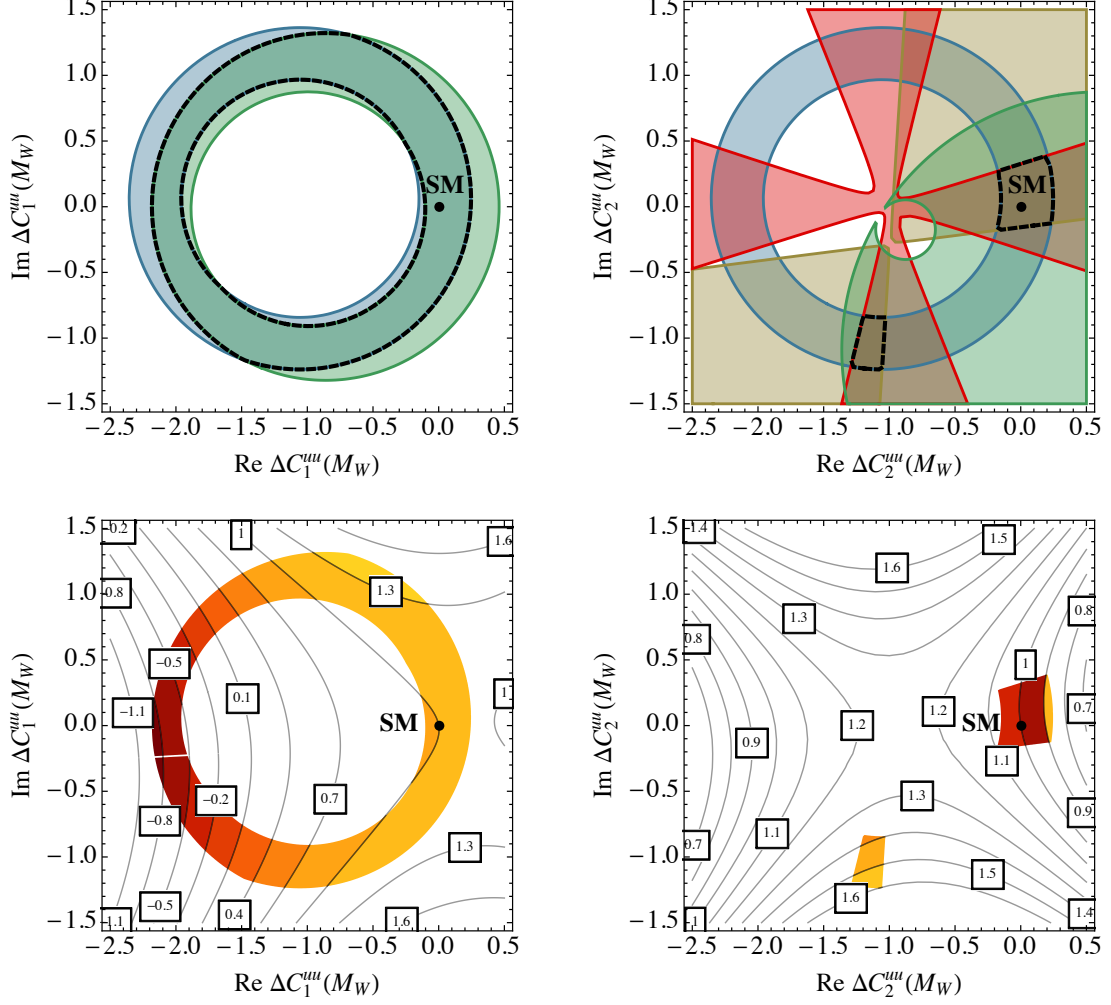
The corresponding experimental value is [21]

$$R(\rho^-\rho^0/\rho^+\rho^-) = 0.89 \pm 0.14. \quad (4.19)$$

The 90% CL constraints from  $R(\rho^-\rho^0/\rho^+\rho^-)$  are shown in the green regions in Figure 1. In the case of  $\Delta C_1^{uu}$  the constraint is only moderately useful, cutting out a small part of the otherwise allowed region. The constraint on  $\Delta C_2^{uu}$ , on the other hand, serves to completely eliminate one of the regions where the phase differs vastly from the SM value.<sup>8</sup>

This concludes our discussion of constraints on the Wilson coefficients  $\Delta C_{1,2}^{uu}$ . In Figure 1 the combined constraints on the Wilson coefficients are overlaid on contours showing the ratio  $\Delta\Gamma_d/\Delta\Gamma_d^{\text{SM}}$ . The allowed region for  $\Delta C_1^{uu}$  contains areas where  $\Delta\Gamma_d$  is enhanced by up to 30% and also those where it is decreased significantly compared to its SM value. The allowed region for  $\Delta C_2^{uu}$  around the SM value does not allow for any significant changes in the ratio  $\Delta\Gamma_d/\Delta\Gamma_d^{\text{SM}}$ , while that where the phase is significantly different contains areas where the ratio is increased by approximately 50% compared to the SM value.

<sup>8</sup>It is worth mentioning that the central value in (4.18) depends rather strongly on the value of the hadronic input parameter  $\lambda_B$ : we use the value of [41] quoted in Table 3. Using lower values of  $\lambda_B$  brings the SM number closer to the experimental result [45].



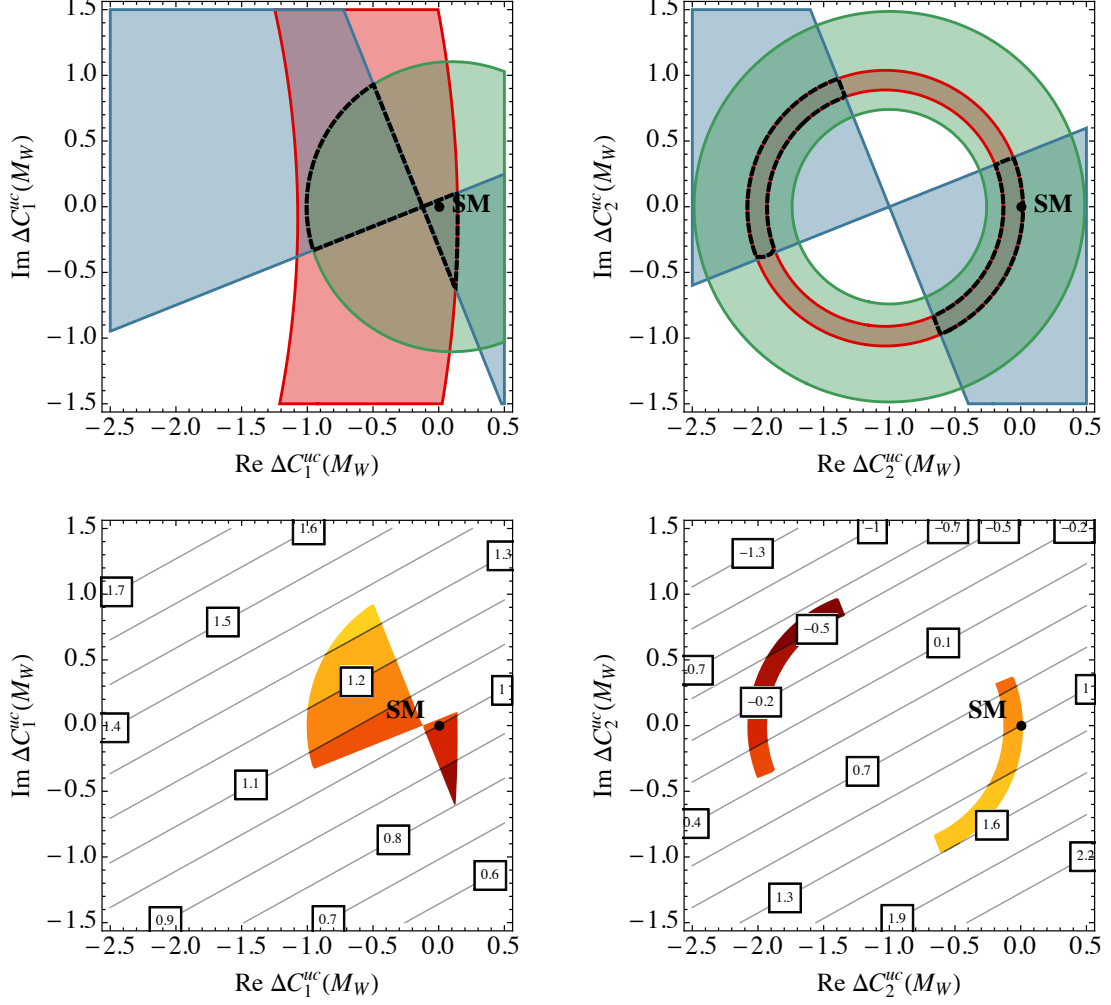
**Figure 1.** Upper panels: allowed parameter space in the  $\text{Re}\Delta C_{1,2}^{uu}-\text{Im}\Delta C_{1,2}^{uu}$  planes. The blue, brown, red, and green regions are related to the constraints derived from  $R_{\pi^-\pi^0}$ ,  $S_{\pi^+\pi^-}$ ,  $S_{\rho\pi}$  and  $R(\rho^-\rho^0/\rho^+\rho^-)$ , respectively. The regions enclosed by the dashed black lines represent the combined constraint from the different observables. Lower panels: contours of  $\Delta\Gamma_d/\Delta\Gamma_d^{\text{SM}}$  along with the combined constraints.

#### 4.2 Bounds on up-charm-quark operators

The Wilson coefficients  $C_{1,2}^{uc}$  multiply the operators governing  $b \rightarrow c\bar{u}d$  transitions. Such operators mediate exclusive decays such as  $B \rightarrow D\pi$  and are also important for the total  $B$ -meson decay rate, which receives its largest contribution from  $b \rightarrow c\bar{u}d$  decays. In this section we consider constraints from these two processes.

We first examine  $B \rightarrow D\pi$  decays. A particularly clean test of the current-current sector can be obtained by relating the  $\bar{B}^0 \rightarrow D^{*+}\pi^-$  decay rate to the differential semi-leptonic  $\bar{B}^0 \rightarrow D^{*+}l^-\bar{\nu}_l$  rate normalised at  $q^2 = m_{\pi^-}^2$ . One has [47]

$$R_{D^{*+}\pi^-} = \frac{\Gamma(\bar{B}^0 \rightarrow D^{*+}\pi^-)}{d\Gamma(\bar{B}^0 \rightarrow D^{*+}l^-\bar{\nu}_l)/dq^2|_{q^2=m_{\pi^-}^2}} \simeq 6\pi^2 f_\pi^2 |V_{ud}|^2 |\alpha_2(D^*\pi)|^2, \quad (4.20)$$



**Figure 2.** Upper panels: allowed parameter space in the  $\text{Re}\Delta C_{1,2}^{uc}$ – $\text{Im}\Delta C_{1,2}^{uc}$  planes. The red regions are related to constraints from  $R_{D^{*+}\pi^-}$ , the green ones from the total  $B$ -meson decay width and the blue ones from indirect CP asymmetries in exclusive  $b \rightarrow c\bar{u}d$  decays. The regions enclosed by the dashed black lines represent the combined constraint from the different observables. Lower panels: contours of  $\Delta\Gamma_d/\Delta\Gamma_d^{\text{SM}}$  along with the combined constraints.

with

$$\alpha_2(D^*\pi) = 1.054 + 0.013i + \Delta C_{1/3}^{uc}. \quad (4.21)$$

The SM value is obtained at NLO from [47], while the new-physics contribution is LO accurate only. After restoring the theoretical error in the SM calculation it follows that

$$R_{D^{*+}\pi^-} \simeq (1.07 \pm 0.04) \left[ 1 + 1.90 \text{Re}\Delta C_{1/3}^{uc} + 0.02 \text{Im}\Delta C_{1/3}^{uc} + 0.90 |\Delta C_{1/3}^{uc}|^2 \right] \text{GeV}^2. \quad (4.22)$$

On the experimental side one has [48]

$$R_{D^{*+}\pi^-} = (0.96 \pm 0.08) \text{GeV}^2. \quad (4.23)$$



In Figure 2 we display in red the 90% CL constraints on  $\Delta C_{1,2}^{uc}(M_W)$  obtained from  $R_{D^{*+}\pi^-}$ . Obviously, the allowed region is the one which leaves the quantity  $|\alpha_2(D^*\pi)|$  SM-like and is thus a circle in the complex plane. Moreover, since the result is sensitive only to the combination  $\Delta C_{1/3}^{uc}(m_b)$ , the constraints on  $\Delta C_1^{uc}(M_W)$  are quite weak, which follows from (4.17).

In addition to the ratio of branching ratios above, we can also consider indirect CP asymmetries in colour-suppressed  $b \rightarrow c\bar{u}d$  transitions. These have been measured, for instance in [49], which gave results for  $B^0 \rightarrow D^{(*)0}h^0$  decays, where  $h^0$  is a  $\pi^0$ ,  $\eta$  or  $\omega$  meson and the  $D^{(*)0}$ -meson decays into a CP eigenstate. In that case the amplitudes which determine the indirect asymmetry through (2.6) are dominated by  $b \rightarrow c\bar{u}d$  transitions, because the contribution from the  $b \rightarrow u\bar{c}d$  is CKM suppressed. In the SM, where the Wilson coefficients  $C_{1,2}^{uc}$  are real, the indirect asymmetry is directly proportional to  $\sin(2\beta)$  when the  $b \rightarrow u\bar{c}d$  transitions are neglected. Beyond the SM the Wilson coefficients governing the decay are in general complex and one must calculate the dependence of the decay amplitudes on  $C_{1,2}^{uc}$  in order to obtain the indirect asymmetry. To the best of our knowledge there are no QCD factorisation predictions for such decay amplitudes. Resorting to an approximation in “naive factorisation”, however, we can estimate the indirect CP asymmetry to be

$$S_{D^{(*)0}h^0} = 2 \frac{\text{Im}(e^{-2i\beta}\rho_{D^{(*)0}h^0})}{1 + |\rho_{D^{(*)0}h^0}|^2}, \quad \rho_{D^{(*)0}h^0} = \frac{C_1^{uc} + C_2^{uc}/3}{(C_1^{uc})^* + (C_2^{uc})^*/3}. \quad (4.24)$$

In this approximation the indirect asymmetry is independent of the decay channel and can be compared with the experimental average from [49]

$$S_{D^{(*)0}h^0} = -(56 \pm 24) \%. \quad (4.25)$$

We wish to combine these two results into bounds on the Wilson coefficients, but in order to do so we must assign an error to the theoretical estimate (4.24). There is no obvious way to do this, so for lack of a better procedure we assign a theory error equal to the experimental one above. The 90% CL bounds on  $\Delta C_{1,2}^{uc}(M_W)$  obtained using these results are shown in the blue regions of Figure 2. We will discuss the utility of these constraints below.

In addition to exclusive decay modes, one can also examine constraints from the total  $B$ -meson decay rate. In fact, the largest contribution to the total decay rate comes from the  $b \rightarrow c\bar{u}d$  contribution, which probes  $\Delta C_{1,2}^{uc}$ . Contributions from the  $b \rightarrow c\bar{c}d$ ,  $b \rightarrow u\bar{u}d$  and  $b \rightarrow u\bar{c}d$  modes are suppressed by at least a factor of 40 compared to this dominant contribution and thus the total decay rate is largely insensitive to changes in the Wilson coefficients associated with these contributions, i.e.  $\Delta C_{1,2}^{cc}$ ,  $\Delta C_{1,2}^{uu}$  and  $\Delta C_{1,2}^{cu}$  respectively. Working to NLO in  $\alpha_s$  and to  $\Lambda_{\text{QCD}}^2/m_b^2$  in the HQE, one finds (based on the numerical evaluation in [33])

$$\Gamma_{\text{tot}}^{\text{SM}} = (3.6 \pm 0.8) \cdot 10^{-13} \text{ GeV}, \quad \frac{\Gamma_{b \rightarrow c\bar{u}d}^{\text{SM}}}{\Gamma_{\text{tot}}^{\text{SM}}} \simeq 44\%. \quad (4.26)$$

We use these numbers to arrive at an approximate result for the total decay rate as a function of the Wilson coefficients  $C_{1,2}^{uc}$ :

$$\frac{\Gamma_{\text{tot}}}{\Gamma_{\text{tot}}^{\text{SM}}} \simeq \left(1 - \frac{\Gamma_{b \rightarrow c\bar{u}d}^{\text{SM}}}{\Gamma_{\text{tot}}^{\text{SM}}}\right) + \frac{3|C_1^{uc}|^2 + 3|C_2^{uc}|^2 + 2\text{Re}[(C_1^{uc})^*C_2^{uc}]}{(3|C_1^{uc}|^2 + 3|C_2^{uc}|^2 + 2\text{Re}[(C_1^{uc})^*C_2^{uc}])_{\text{SM}}} \frac{\Gamma_{b \rightarrow c\bar{u}d}^{\text{SM}}}{\Gamma_{\text{tot}}^{\text{SM}}}. \quad (4.27)$$

The approximation is obtained by singling out the combination of Wilson coefficients which multiplies the  $b \rightarrow c\bar{u}d$  decay rate at LO in the HQE and is thus valid to that accuracy. The experimental result is [21]

$$\Gamma_{\text{tot}} = (4.20 \pm 0.02) \cdot 10^{-13} \text{ GeV}. \quad (4.28)$$

The 90% CL bounds on  $\Delta C_{1,2}^{uc}(M_W)$  resulting from the above equations for the total  $B$ -meson decay width are shown in green in Figure 2. It is evident that the total decay rate adds no further constraint on the coefficient  $\Delta C_2^{uc}$  compared to that from the exclusive decays discussed above. On the other hand, it is quite useful in constraining  $\Delta C_1^{uc}$ , since it singles out the area of parameter space where the phase is SM-like.

The lower panels of Figure 2 show the combined 90% CL constraints on  $\Delta C_{1,2}^{uc}(M_W)$  derived above, overlaid on contours of the ratio  $\Delta\Gamma_d/\Delta\Gamma_d^{\text{SM}}$ . In the allowed region for  $\Delta C_1^{uc}$ ,  $\Delta\Gamma_d$  can be increased (decreased) by around 30% (20%) compared to its SM value. In the allowed region for  $\Delta C_2^{uc}$ , on the other hand,  $\Delta\Gamma_d$  can be increased by up to 60%, but can also be decreased and even reach negative values. These statements are independent of whether constraints from the indirect CP asymmetries, which are on weaker theoretical ground than the other two constraints, are included in the analysis.

### 4.3 Bounds on charm-charm-quark operators

The Wilson coefficients  $C_{1,2}^{cc}$  multiply the operators governing  $b \rightarrow c\bar{c}d$  transitions. These coefficients are especially important for  $\Delta\Gamma_d$ , since they determine the contribution to  $\Gamma_{12}^d$  involving two charm quarks, which as seen from (3.3) dominates the real part of this quantity. However, the  $b \rightarrow c\bar{c}d$  quark decay provides only a negligible contribution to the total  $B$ -meson decay width and branching ratios for decays into hadronic final states are under poor control theoretically. This situation leads us to investigate indirect constraints from  $B \rightarrow X_d\gamma$  decays and the restrictions imposed by the dimension-eight contributions to  $\sin(2\beta)$ . We also discuss indirect CP asymmetries in exclusive  $b \rightarrow c\bar{c}d$  transitions and the semi-leptonic asymmetry  $a_{sl}^d$ .

We begin by considering  $B \rightarrow X_d\gamma$ . The branching ratio for this decay is sensitive to the Wilson coefficients  $C_{1,2}^{cc}(M_W)$  even at leading logarithmic (LL) order in perturbation theory, due to operator mixing. To clarify this point, we follow [50] and write

$$\text{Br}(B \rightarrow X_d\gamma)_{E_\gamma > E_0} = \mathcal{N}_{B \rightarrow X_d\gamma} \left| \frac{\lambda_t^d}{V_{cb}} \right|^2 [P(E_0) + N(E_0)], \quad (4.29)$$

where  $\mathcal{N}_{B \rightarrow X_d\gamma} = 2.57 \cdot 10^{-3}$  is a normalisation factor,  $P(E_0)$  is proportional to the perturbative expression for the partonic decay  $b \rightarrow X_d\gamma$ ,  $N(E_0)$  denotes power-suppressed nonperturbative contributions and  $E_0$  represents a cut on the photon energy  $E_\gamma$ . At LL accuracy the perturbative contribution is given by  $P(E_0) = |C_7^{(0)}(m_b)|^2$ , where  $C_7$  is the Wilson coefficient of the electromagnetic dipole operator (see (5.4) below). In terms of matching coefficients at the scale  $M_W$ , one finds

$$C_7^{(0)}(m_b) \simeq -0.31 - 0.17 \Delta C_2^{cc}(M_W). \quad (4.30)$$

The following approximation for the branching ratio is thus valid

$$\begin{aligned} \text{Br}(B \rightarrow X_d \gamma)_{E_\gamma > E_0} &\simeq \text{Br}(B \rightarrow X_d \gamma)_{E_\gamma > E_0}^{\text{SM}} \\ &+ \frac{\mathcal{N}_{B \rightarrow X_d \gamma}}{10} \left| \frac{\lambda_t^d}{V_{cb}} \right|^2 \left( 1.1 \text{Re} \Delta C_2^{cc}(M_W) + 0.3 |\Delta C_2^{cc}(M_W)|^2 \right). \end{aligned} \quad (4.31)$$

We can obtain bounds on the Wilson coefficient  $\Delta C_2^{cc}(M_W)$  using the above equations, together with the recent SM prediction for the CP-averaged branching ratio [51]

$$\text{Br}(B \rightarrow X_d \gamma)_{E_\gamma > 1.6 \text{ GeV}}^{\text{SM}} = (1.54_{-0.31}^{+0.26}) \cdot 10^{-5}, \quad (4.32)$$

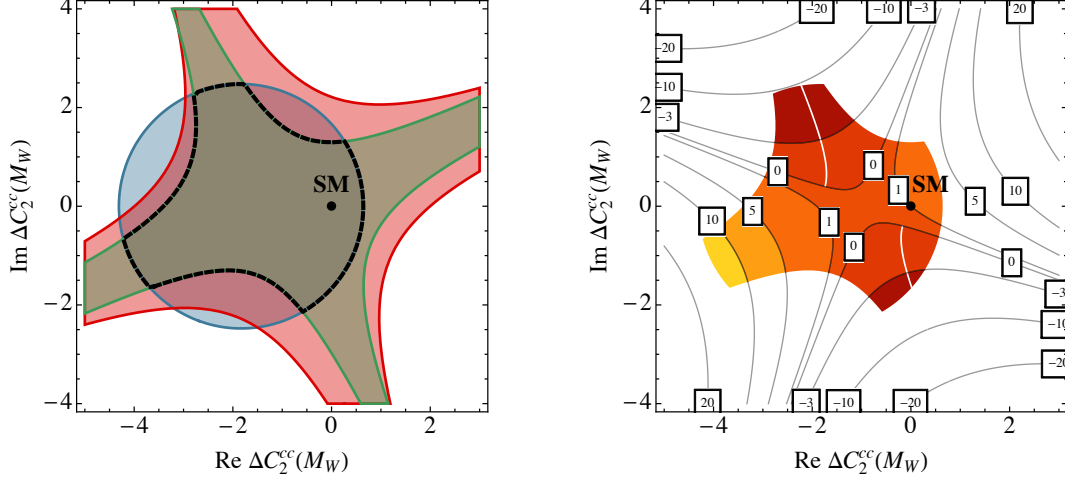
and the latest experimental result [51, 52]

$$\text{Br}(B \rightarrow X_d \gamma)_{E_\gamma > 1.6 \text{ GeV}} = (1.41 \pm 0.57) \cdot 10^{-5}. \quad (4.33)$$

The 90% CL bound on  $\Delta C_2^{cc}(M_W)$  resulting from this analysis is shown by the blue circle in Figure 3. Obviously, the LL expression (4.31) does not translate into severe constraints. It could be slightly improved by noting that even though the correction to the  $B \rightarrow X_d \gamma$  decay rate from the matrix element of  $Q_2^{cc}$  first appears at NLO in  $\alpha_s$ , it is nonetheless sizable due to the fact that the SM coefficient  $C_2^{\text{SM}}(m_b) \simeq 1.1$  is about 4 times larger than  $C_7^{\text{SM}}(m_b) \simeq -0.31$  in magnitude. Moreover, the virtual corrections from this operator are more important than those related to real emission. We have examined how the constraints shown in the figure change when adding on these NLO virtual corrections to (4.31). It turns out that the allowed region is moderately reduced, but not to the degree that it would change the qualitative analysis, so we do not discuss modifications of (4.31) due to higher-order QCD corrections any further.

The results from  $B \rightarrow X_d \gamma$  branching ratio are of little use for constraining potential new-physics contributions to  $\Delta \Gamma_d$ , as is verified by comparing the allowed region in the blue circle with the contours of  $\Delta \Gamma_d / \Delta \Gamma_d^{\text{SM}}$  in the right-hand panel of Figure 3. We have checked that even weaker constraints arise from  $A_{\text{CP}}(B \rightarrow X_{d+s} \gamma)$ ,  $B^+ \rightarrow \pi^+ \mu^+ \mu^-$  and the  $B$ -meson lifetime. In principle, we could also consider indirect asymmetries in exclusive  $b \rightarrow c \bar{c} d$  decays, analogous to those for the  $b \rightarrow c \bar{u} d$  transitions discussed above. However, in the present case the difficulties in estimating theory uncertainties related to hadronic matrix elements are compounded by the presence of penguin amplitudes carrying a different weak phase to the current-current contributions. We have checked that even being relatively aggressive with theory errors yields a constraint broadly similar to the blue triangular regions in Figure 2, which in any case does not help in ruling out areas of parameter space where  $\Delta \Gamma_d$  is dramatically different from the SM value. For these reasons, we do not use these indirect asymmetries in our analysis.

The situation described above leads us to seek additional experimental constraints on the  $\Delta C_i^{cc}$  coefficients from observables in the  $B_d$ -meson mixing system itself. One such constraint arises from the precise determination of  $\sin(2\beta)$  (see (2.7)) from the time-dependent CP asymmetry in  $B \rightarrow J/\psi K_S$  decays. Within the SM the latter observable is to excellent precision simply given by the imaginary part of the dimension-six contribution



**Figure 3.** Left panel: allowed parameter space in the  $\text{Re}\Delta C_2^{cc}-\text{Im}\Delta C_2^{cc}$  planes. The blue circular region is related to constraints from  $B \rightarrow X_d \gamma$ , and the green region from those from semi-leptonic asymmetry  $a_{sl}^d$ , and the red region from the dimension-eight contributions to  $\sin(2\beta)$ . The region enclosed by the dashed black lines represent the combined constraint from the different observables. Right panel: contours of  $\Delta\Gamma_d/\Delta\Gamma_d^{\text{SM}}$ .

to  $M_{12}^d$  that arises from box diagrams with top-quark and  $W$ -boson exchange. Additional dimension-eight contributions stem from double insertions of  $\Delta B = 1$  operators such as the current-currents operators. Including both types of corrections, but restricting ourselves for the moment to insertions of  $Q_{1,2}^{cc}$ , we obtain

$$M_{12}^d \simeq \frac{G_F^2}{12\pi^2} M_{B_d} f_{B_d}^2 B_V \left\{ (\lambda_t^d)^2 m_t^2 K_6 + (\lambda_c^d)^2 m_b^2 \left[ (C_2^{cc})^2 K_8^{cc} + \left( 2C_2^{cc} C_1^{cc} + 3(C_1^{cc})^2 \right) \tilde{K}_8^{cc} \right] \ln \left( \frac{M_W^2}{m_b^2} \right) \right\}. \quad (4.34)$$

Here and in the remainder of the section all Wilson coefficients are to be evaluated at the scale  $\mu = M_W$ . After employing  $C_2^{cc} \simeq 1 + \Delta C_2^{cc}$  and  $C_1^{cc} \simeq \Delta C_1^{cc}$  we confirm the results of [18] for the SM contribution. The coefficient that multiplies the dimension-six contribution is given at next-to-leading logarithmic (NLL) accuracy by  $K_6 \simeq 0.47$ , while we left the leading logarithm of the dimension-eight contribution unresummed, which is sufficient for our purposes. The corresponding dimension-eight coefficients take the form

$$K_8^{cc} = \frac{1}{3} - \frac{2m_c^2}{m_b^2} + \frac{5B_S}{12B_V} \simeq 0.82, \quad \tilde{K}_8^{cc} = \frac{1}{3} - \frac{2m_c^2}{m_b^2} - \frac{\tilde{B}_S}{12B_V} \simeq -1.1 \cdot 10^{-3}, \quad (4.35)$$

where we have used the central values as given in Table 3 of the so-called bag parameters to obtain the final numerical values. Varying  $B_V$ ,  $B_S$  and  $\tilde{B}_S$  within their uncertainties we find that  $|\tilde{K}_8^{cc}/K_8^{cc}| < 0.05$ , which implies that the contribution to  $M_{12}^d$  involving  $C_1^{cc}$  is generically suppressed by more than an order of magnitude. The dimension-eight contribution involving only up quarks is obtained from (4.34) and (4.35) by replacing  $(\lambda_c^d)^2 \rightarrow (\lambda_u^d)^2$ ,

$C_{1,2}^{cc} \rightarrow C_{1,2}^{uu}$  and  $m_c^2 \rightarrow 0$ , while in the case of the charm-up-quark contribution one has to employ  $(\lambda_c^d)^2 \rightarrow 2\lambda_c^d \lambda_u^d$ ,  $(C_2^{cc})^2 \rightarrow C_2^{cu} C_2^{uc}$ ,  $2C_2^{cc} C_1^{cc} + 3(C_1^{cc})^2 \rightarrow C_2^{cu} C_1^{uc} + C_2^{uc} C_1^{cu} + 3C_1^{cu} C_1^{uc}$  and  $m_c^2 \rightarrow m_c^2/2$ . Notice that since  $m_c^2/m_b^2 \simeq 0.1$  and  $K_8^{pp'} \gg \tilde{K}_8^{pp'}$  the coefficients  $C_2^{pp'}$  are for each flavour combination much stronger constrained by the measurement of the time-dependent CP asymmetry in  $B \rightarrow J/\psi K_S$  than  $C_1^{pp'}$ .

The correction  $\phi_d^\Delta$  to the  $B_d$ -mixing phase  $2\beta$  is related to the off-diagonal element  $M_{12}^d$  of the mass matrix via

$$\sin(2\beta + \phi_d^\Delta) = \text{Im} \left( \frac{M_{12}^d}{|M_{12}^d|} \right). \quad (4.36)$$

We can obtain a simple analytic expression for the new-physics phase by approximating  $\lambda_t^d \simeq |\lambda_t^d| e^{i\beta}$ ,  $\lambda_c^d \simeq -|\lambda_c^d|$  and  $|\lambda_c^d/\lambda_t^d| \simeq 1$ , expanding in powers of  $m_b^2/m_t^2$ , ignoring contributions proportional to  $\tilde{K}_8^{cc}$  and neglecting the tiny SM contribution to  $\phi_d^\Delta$  of  $\mathcal{O}(10^{-4})$  [18]. We then find from (4.34)

$$\phi_d^\Delta \simeq -\frac{m_b^2}{m_t^2} \frac{K_8^{cc}}{K_6} \left[ \sin(2\beta) \text{Re}(C_2^{cc}) - \cos(2\beta) \text{Im}(C_2^{cc}) \right] \ln \left( \frac{M_W^2}{m_b^2} \right). \quad (4.37)$$

By using in addition  $\lambda_u^d = -\lambda_t^d - \lambda_c^d$  similar results can be shown to hold in the case of the charm-up-quark and up-up-quark contribution.

Current experimental and SM results for the CKM angle  $\beta$  can be found in the last row of Table 1. We would like to use these to constrain new-physics contributions to  $\sin(2\beta)$ , but run into the problem that the experimental and SM results are in poor agreement. One way to deal with this would be to bridge the gap through the new-physics part of the dimension-eight contributions above. However, it is not clear that this discrepancy merits a new-physics explanation. To derive 90% CL constraints from this observable, we instead use the following procedure. First, we ignore the difference in central values of the SM and experimental numbers, and add the experimental and theory errors together in quadrature to derive a total error. We then saturate the total error at 90% CL with the new-physics contribution from (4.36). This procedure leads to the constraint on  $\Delta C_2^{cc}$  shown by the red region in the left panel in Figure 3, which we have obtained using the exact numerical evaluation of (4.34) and (4.36) rather than the approximation (4.37). As mentioned above, the constraint on  $\Delta C_1^{cc}$  is very weak, as are those on the coefficients  $\Delta C_{1,2}^{uu}$  and  $\Delta C_{1,2}^{uc}$ , so we omit these from the discussion. We see that the constraint derived from the dimension-eight contribution to  $\sin(2\beta)$  does very little to eliminate the allowed parameter space. In fact, this is an important result in its own right — if these corrections had turned out to be restrictive, there would be little justification for ignoring dimension-eight operators in other parts of our analysis.

We can derive a more useful restriction by studying the semi-leptonic asymmetry  $a_{sl}^d$ . The new-physics contributions to this quantity can be calculated in a straightforward way using the parameterisation (2.3), including the new-physics contributions related to  $M_{12}^d$  in (4.34) as well as those from (4.4). The recent experimental results from direct measurements of semi-leptonic decays have been given in Table 1. The numbers are consistent

with zero within the large experimental errors, so for simplicity we use as an experimental value  $(0 \pm 1) \%$  in deriving bounds on the Wilson coefficients. The allowed region resulting from this procedure is shown in blue in Figure 3. While some of the allowed space from  $B \rightarrow X_d \gamma$  is cut out upon including this constraint,  $\mathcal{O}(10)$  contributions  $\Delta\Gamma_d$  due to the Wilson coefficient  $\Delta C_2^{cc}$  are not ruled out from present experimental data, as is shown in the right-hand panel of the figure.

## 5 New physics in $\Delta\Gamma_d$ : $(\bar{d}b)(\bar{\tau}\tau)$ operators

In this section we study possible effects on  $\Delta\Gamma_d$  related to effective operators of the form  $(\bar{d}b)(\bar{\tau}\tau)$ . The analogous operators for the  $B_s$ -meson system (i.e. those obtained by replacing  $d \rightarrow s$ ) were introduced in [15] and used in studying  $\Delta\Gamma_s$ . The corresponding effective Hamiltonian reads

$$\mathcal{H}_{\text{eff}}^{b \rightarrow d\tau^+\tau^-} = -\frac{4G_F}{\sqrt{2}} \lambda_t^d \sum_{i,j} C_{i,j}(\mu) Q_{i,j}, \quad (5.1)$$

and involves the following complete set of operators

$$\begin{aligned} Q_{S,AB} &= (\bar{d} P_A b)(\bar{\tau} P_B \tau), \\ Q_{V,AB} &= (\bar{d} \gamma^\mu P_A b)(\bar{\tau} \gamma_\mu P_B \tau), \\ Q_{T,A} &= (\bar{d} \sigma^{\mu\nu} P_A b)(\bar{\tau} \sigma_{\mu\nu} P_B \tau), \end{aligned} \quad (5.2)$$

where  $P_{L,R} = (1 \mp \gamma_5)/2$  and  $A, B = L, R$ . In addition to these operators, our analysis will use the dimension-six effective Hamiltonian describing  $b \rightarrow d\ell^+\ell^-$  transitions ( $\ell = e, \mu, \tau$ ). We write this as

$$\mathcal{H}_{\text{eff}}^{b \rightarrow d\ell^+\ell^-} = -\frac{4G_F}{\sqrt{2}} \lambda_t^d \sum_i C_i(\mu) Q_i. \quad (5.3)$$

The most important operators in what follows are

$$\begin{aligned} Q_7 &= \frac{e}{(4\pi)^2} m_b (\bar{d} \sigma^{\mu\nu} P_R b) F_{\mu\nu}, \\ Q_9 &= \frac{e^2}{(4\pi)^2} (\bar{d} \gamma^\mu P_L b) (\bar{\ell} \gamma_\mu \ell), \\ Q_{10} &= \frac{e^2}{(4\pi)^2} (\bar{d} \gamma^\mu P_L b) (\bar{\ell} \gamma_\mu \gamma_5 \ell), \end{aligned} \quad (5.4)$$

and their chirality-flipped counterparts  $Q'_i$  obtained through the interchange  $P_L \leftrightarrow P_R$ .

The operators (5.2) are interesting because they can give large contributions to  $\Delta\Gamma_d$ , but are only weakly constrained by experimental data. We will see that in the case of the  $B_d$ -meson system the various direct and indirect bounds on the Wilson coefficients of the operators (5.2) are generally weaker than in the  $B_s$ -meson system and that large enhancements of  $\Delta\Gamma_d$  due to such operators are not yet ruled out. We derive direct bounds, where the operators contribute to tree-level matrix elements, from the decays  $B_d \rightarrow \tau^+\tau^-$ ,  $B \rightarrow X_d \tau^+\tau^-$  and  $B^+ \rightarrow \pi^+\tau^+\tau^-$ . Indirect bounds, where the operators contribute either through operator mixing and loop-level matrix elements, are based on  $B \rightarrow X_d \gamma$  and  $B^+ \rightarrow \pi^+\mu^+\mu^-$  decays. We discuss the two cases in turn.

## 5.1 Direct bounds

We first investigate direct constraints from the decay  $B_d \rightarrow \tau^+ \tau^-$ . At present, the only experimental information on this decay is the 90% CL bound from [53]:

$$\text{Br}(B_d \rightarrow \tau^+ \tau^-) < 4.1 \cdot 10^{-3}. \quad (5.5)$$

The theory prediction, including both SM and the effects of the operators (5.2) can be extracted from [54]. The result depends on the SM coefficient  $C_{10}$  in addition to the Wilson coefficients  $C_{S,AB}$  and  $C_{V,AB}$ , but not on the tensor coefficients  $C_{T,A}$ . Moreover, due to a loop suppression factor of  $\alpha/\pi$ , the SM contribution alone is quite small,  $\text{Br}(B_d \rightarrow \tau^+ \tau^-)_{\text{SM}} \simeq 2.3 \cdot 10^{-8}$  [55]. We therefore neglect it in obtaining bounds on the coefficients of the new operators. Furthermore, as before we assume the dominance of a single operator at a time, neglecting interference terms of the new operators both with the SM contribution and with themselves. Following this procedure, we can set bounds on the absolute values of the coefficients  $C_{S,AB}$  and  $C_{V,AB}$ , independent of the chirality structure. Giving up this assumption, would lead to considerably larger bounds on  $\Delta\Gamma_d$ . In that respect our estimates are very conservative.

The branching ratio depends on a number of input parameters. Some of these are common to the other decays discussed in this section: for these we use the values of  $G_F$ ,  $M_{B_d}$ ,  $\tau_{B_d}$ ,  $f_{B_d}$ ,  $|\lambda_d^t|$ ,  $m_\tau$ ,  $\alpha(M_Z)$ ,  $m_b^{\text{pole}}$  and  $m_d$  summarised in Table 3. We then obtain at  $m_b = \bar{m}_b(\bar{m}_b) \simeq 4.2$  GeV the results  $|C_{S,AB}(m_b)| < 1.1$  and  $|C_{V,AB}(m_b)| < 2.2$ , which are also collected in Table 2. These are the strongest bounds on the scalar and vector coefficients that will emerge from our analysis.

We next consider inclusive and exclusive  $b \rightarrow d\tau^+ \tau^-$  decays. In this case, there are no direct experimental bounds on the branching ratios. However, we can use information from the  $B_{d,s}$ -meson lifetimes to estimate the potential size of new-physics contributions to these decays. We first note that the SM prediction for the lifetime ratio is very close to unity [4]

$$\left( \frac{\tau_{B_s}}{\tau_{B_d}} - 1 \right)_{\text{SM}} = (-0.2 \pm 0.2) \%. \quad (5.6)$$

Current experimental measurements [21] are compatible with this prediction:

$$\left( \frac{\tau_{B_s}}{\tau_{B_d}} - 1 \right) = (-0.2 \pm 0.9) \%. \quad (5.7)$$

Comparing the results, one can get a rough bound on the size of possible new-physics contributions  $\Gamma_{d,s}^{\text{NP}}$  to the total  $B_{d,s}$ -meson decay rates  $\Gamma_{d,s}$ , namely

$$\frac{\Gamma_d^{\text{NP}} - \Gamma_s^{\text{NP}}}{\Gamma_s} = (0.0 \pm 0.9) \%. \quad (5.8)$$

Setting  $\Gamma_s^{\text{NP}} = 0$  gives an upper bound on (also invisible) new-physics contributions to  $B_d$ -meson decays. At 90% CL one obtains

$$\text{Br}(B_d \rightarrow X) \leq 1.5\%. \quad (5.9)$$



We now turn this estimate into bounds on Wilson coefficients using the theoretical expressions for the  $B \rightarrow X_d \tau^+ \tau^-$  and  $B^+ \rightarrow \pi^+ \tau^+ \tau^-$  branching ratios. In contrast to  $B_d \rightarrow \tau^+ \tau^-$  decays, in these cases all operators contribute, so we also gain information on the tensor coefficients. However, once again the results are independent of the chirality structure and allow us to constrain only absolute values of the coefficients. For the inclusive decay  $B \rightarrow X_d \tau^+ \tau^-$ , we use the expressions for the branching ratios given in Section 3 and the appendix of [15] after appropriate modifications. Most of the inputs to the branching ratio are common to  $B_d$ -meson and  $B_s$ -meson decays. Apart from trivial differences related to meson masses and lifetimes (for the exclusive decay we use  $\tau_{B^+}$  and the CKM factor  $|V_{td}^* V_{tb}|/|V_{cb}|$  given in Table 3). The exclusive decay  $B^+ \rightarrow \pi^+ \tau^+ \tau^-$  depends on these parameters and also three  $B \rightarrow \pi$  form factors ( $f_{+,T,0}$ ), as a function of the dilepton invariant mass, denoted as  $q^2$ . For these we use the results of [56].<sup>9</sup> Moreover, we integrate over the full kinematic range  $q^2 \in [4m_\tau^2, (M_{B^+} - M_{\pi^+})^2]$ .

We must also decide on a value for the experimental branching ratios. At present, we can only use (5.9) to estimate the size of the  $B \rightarrow X_d \tau^+ \tau^-$  and  $B^+ \rightarrow \pi^+ \tau^+ \tau^-$  branching ratios, which most likely overestimates their allowed ranges. Here a direct experimental bound would be very helpful. For reference, we collect the obtained bounds on the Wilson coefficients using the 90% CL estimates in Table 2. In Section 5.3, we will show the size of possible enhancement of  $\Delta\Gamma_d$  as a function of measured branching ratios. Compared to the bounds on the  $B \rightarrow X_d \tau^+ \tau^-$  and  $B^+ \rightarrow \pi^+ \tau^+ \tau^-$  branching ratios estimated through (5.9), we find tiny SM predictions

$$\begin{aligned} \text{Br}(B \rightarrow X_d \tau^+ \tau^-)_{\text{SM}} &= (1.2 \pm 0.3) \cdot 10^{-8}, \\ \text{Br}(B^+ \rightarrow \pi^+ \tau^+ \tau^-)_{\text{SM}} &= (1.5 \pm 0.5) \cdot 10^{-8}. \end{aligned} \tag{5.10}$$

In both cases our results refer to the fully-integrated and non-resonant branching ratios. The inclusive decay includes just the LO corrections, but accounts for contributions proportional to the tau mass that are of kinematic origin [58]. As for the exclusive mode, we stayed within naive factorisation and the error reflects the uncertainty due to the use of different  $B \rightarrow \pi$  form-factor determinations [56, 57].

## 5.2 Indirect bounds

Indirect bounds arise from cases where the operators (5.2) do not give tree-level contributions to the decays, but contribute either through operator mixing, through loop-level matrix elements or through both. The theoretical expressions needed to set various indirect bounds can be adapted from [15]. It turns out that the most stringent indirect bounds on the Wilson coefficients can be derived from  $B \rightarrow X_d \gamma$  and  $B^+ \rightarrow \pi^+ \mu^+ \mu^-$  decays. We have also examined constraints from  $B_d \rightarrow \gamma \gamma$  decays, but these are rather weak compared to the other decays and in some cases they depend very strongly on hadronic input parameters, so we do not discuss them further.

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<sup>9</sup>We have also derived bounds using the form factors from [57], which yields similar but slightly more stringent bounds.



We first derive bounds from  $B \rightarrow X_d \gamma$  decays. The structure of branching ratios for these decays has been discussed in Section 4.3. We find that the quantity  $P(E_0)$  in (4.29) can be written as

$$P(E_0) = \left| C_7^{\text{SM}(0)}(m_b) + \Delta C_7^{\text{eff}}(m_b) \right|^2 + \left| \Delta C_7^{\text{eff}'}(m_b) \right|^2, \quad (5.11)$$

where

$$\Delta C_7^{\text{eff}(\prime)}(m_b) = \sqrt{x_\tau} (0.62 - 1.09 \eta_6^{-1} + 4 \ln x_\tau) C_{T,R(L)}(m_b). \quad (5.12)$$

We have defined  $\sqrt{x_\tau} \equiv m_\tau/m_b^{\text{pole}}$  and the quantity  $\eta_6 \equiv \alpha_s(\Lambda_{\text{NP}})/\alpha_s(m_t)$ , where  $\alpha_s$  is to be evaluated with six active flavours. The first two terms in (5.12) arise from operator mixing and the third is the matrix element of  $Q_{T,A}$ . In order to derive 90% CL constraints on the Wilson coefficients  $C_{T,A}(m_b)$ , we insert (5.11) into (4.29) and compare with the experimental result (4.33), as usual considering the dominance of one operator at a time. This procedure yields for  $\Lambda_{\text{NP}} \simeq 1$  TeV the bounds  $|C_{T,R}(m_b)| < 0.2$  and  $|C_{T,L}(m_b)| < 0.1$ , which translate to  $|\Delta C_7^{\text{eff}}(m_b)| < 0.7$  and  $|\Delta C_7^{\text{eff}'}(m_b)| < 0.3$ .

The rare decay  $B^+ \rightarrow \pi^+ \mu^+ \mu^-$  has been observed by LHCb [59] in the 2011 data sample of  $1 \text{ fb}^{-1}$  with  $(25.3_{-6.4}^{+6.7})$  events. This provides the first measurement of the non-resonant branching ratio

$$\text{Br}(B^+ \rightarrow \pi^+ \mu^+ \mu^-) = (2.3 \pm 0.6) \cdot 10^{-8}. \quad (5.13)$$

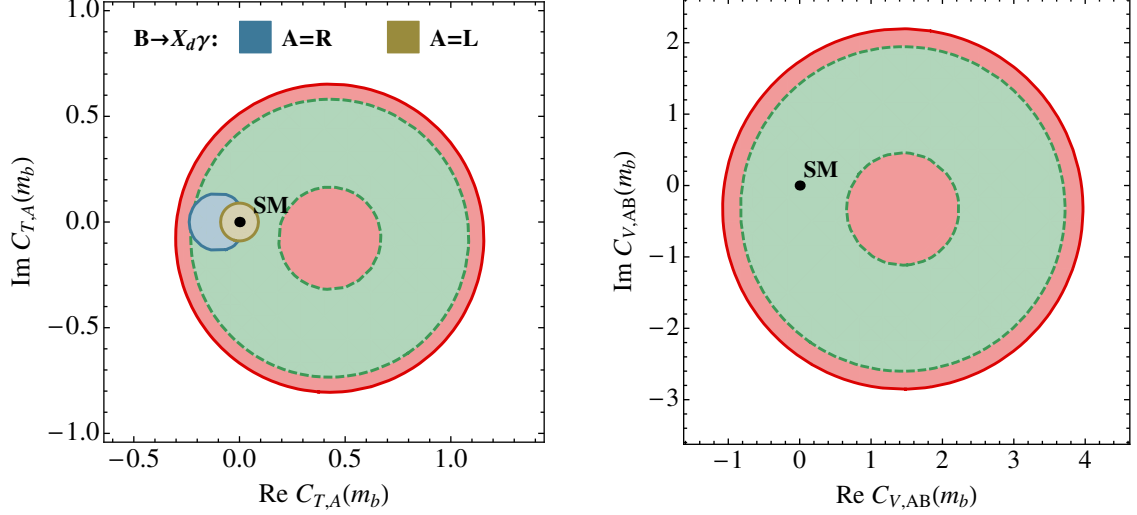
integrated over the whole dilepton invariant mass range  $q^2 \in [4m_\mu^2, (M_{B^+} - M_{\pi^+})^2]$ .

In principle, the calculation of exclusive  $B \rightarrow M \ell^+ \ell^-$  ( $M = P, V$ ) decays is advanced, see [60] for the most recent prediction of  $B^+ \rightarrow \pi^+ \mu^+ \mu^-$  in the SM, where  $B \rightarrow \pi \ell \nu_\ell$  data has been used to extract information on the form factors. The inclusion of corrections beyond naive factorisation in QCD factorisation at low  $q^2$  has been discussed in [61] (see also [62]), whereas at high  $q^2$  a local operator product expansion can be applied to account for resonant contributions [63, 64]. However, in the absence of experimental measurements for either region separately and in view of the large experimental uncertainty, we evaluate the branching ratio in the naive factorisation approximation following [65]. Employing the  $B \rightarrow \pi$  form factors of [56], we obtain

$$\text{Br}(B^+ \rightarrow \pi^+ \mu^+ \mu^-)_{\text{SM}} = (2.1 \pm 0.4) \cdot 10^{-8}. \quad (5.14)$$

The given uncertainty encodes the error related to differences in the existing  $B \rightarrow \pi$  form-factor determinations [56, 57]. Our prediction (5.14) is close both to the measured value (5.13) and the SM value obtained in [60].

Like in the case of the inclusive decay  $B \rightarrow X_d \gamma$ , the effective operators (5.2) contribute to  $B^+ \rightarrow \pi^+ \mu^+ \mu^-$  via mixing into the operators mediating  $b \rightarrow d \gamma$  and  $b \rightarrow d \ell^+ \ell^-$  ( $\ell = e, \mu$ ). The case of  $b \rightarrow s$  transitions has been previously discussed in [15] and can be adopted with appropriate replacements to  $b \rightarrow d$  transitions. One then finds contributions from the tensor coefficients  $C_{T,A}$ , and also on the linear combination  $(C_{V,AL} + C_{V,AR})$  of the vector coefficients. The scalar Wilson coefficients  $C_{S,AB}$  are not subject to constraints from  $B^+ \rightarrow \pi^+ \mu^+ \mu^-$ .



**Figure 4.** The 90% CL regions of  $C_{T,A}(m_b)$  (left) and  $(C_{V,AL}(m_b) + C_{V,AR}(m_b))$  (right) from  $\text{Br}(B^+ \rightarrow \pi^+ \mu^+ \mu^-)$  (red) and  $\text{Br}(B \rightarrow X_d \gamma)$  for  $T, A = T, R$  (blue) and  $T, A = T, L$  (brown). The prospects assuming a measurement of  $\text{Br}(B^+ \rightarrow \pi^+ \mu^+ \mu^-)$  with  $7 \text{ fb}^{-1}$  at LHCb are shown as dashed (green) contours.

In Figure 4, we show the 90% CL regions in the planes of complex-valued  $C_{T,A}(m_b)$  and  $(C_{V,AL}(m_b) + C_{V,AR}(m_b))$  allowed by  $\text{Br}(B^+ \rightarrow \pi^+ \mu^+ \mu^-)$ . In the plots a theory uncertainty of 20% of the SM prediction is assumed and the form-factor predictions [56] are used. We see that in the case of the tensor Wilson coefficients,  $B^+ \rightarrow \pi^+ \mu^+ \mu^-$  provides at present the constraint  $|C_{T,A}(m_b)| \lesssim 1.2$ , which as indicated is clearly weaker than the sensitivity of the inclusive decay  $B \rightarrow X_d \gamma$ . Assuming single operator dominance, the current constraint on the vector Wilson coefficients  $|C_{V,AB}(m_b)| \lesssim 4.0$  is not as strong as the one from  $B_d \rightarrow \tau^+ \tau^-$ . For comparison we also show contours assuming that LHCb has collected  $7 \text{ fb}^{-1}$  of data by 2017. For this purpose the current statistical errors have been rescaled by a factor  $1/\sqrt{7}$ . This exercise shows the potential of this decay mode to reduce further the allowed ranges of  $b \rightarrow d \tau^+ \tau^-$  Wilson coefficients. Depending on the central value of the measurement, it will provide complementary constraints to  $B \rightarrow X_d \gamma$  for the tensor Wilson coefficients.

### 5.3 Maximal effects in width difference

We now explore the consequences of the bounds on the Wilson coefficients (5.1) obtained in the previous section on the size of possible enhancements in  $\Delta \Gamma_d$ . To do so we consider the parameter  $|\tilde{\Delta}_d|$  introduced in (2.3). For the  $B_s$ -meson case expressions for this quantity as a function of the relevant Wilson coefficients were presented in [15], and we can make use of these results after a trivial substitution of CKM factors. Assuming single operator

Constraint	$ C_{S,AB}(m_b) $	$ C_{V,AB}(m_b) $	$ C_{T,A}(m_b) $
direct			
$\text{Br}(B_d \rightarrow \tau^+\tau^-)$	1.1	2.2	—
$\text{Br}(B \rightarrow X_d\tau^+\tau^-)$	10.6	5.3	1.5
$\text{Br}(B^+ \rightarrow \pi^+\tau^+\tau^-)$	5.9	6.2	2.9
indirect			
$\text{Br}(B \rightarrow X_d\gamma)$	—	—	0.2 for $A = R$ 0.1 for $A = L$
$\text{Br}(B^+ \rightarrow \pi^+\mu^+\mu^-)$	—	4.0	1.2

**Table 2.** Summary of direct and indirect bounds on the Wilson coefficients (5.1) at the bottom-quark mass scale  $m_b = \bar{m}_b(\bar{m}_b) \simeq 4.2$  GeV. The constraint from  $B_d \rightarrow \tau^+\tau^-$  decay follows from the experimental 90% CL bound  $\text{Br}(B_d \rightarrow \tau^+\tau^-) < 4.1 \cdot 10^{-3}$ , whereas those from  $B \rightarrow X_d\tau^+\tau^-$  and  $B^+ \rightarrow \pi^+\tau^+\tau^-$  refer to the 90% CL estimate from (5.9). Note that the bounds are independent of the chiral structure  $A, B = L, R$  unless explicitly indicated.

dominance, we then find

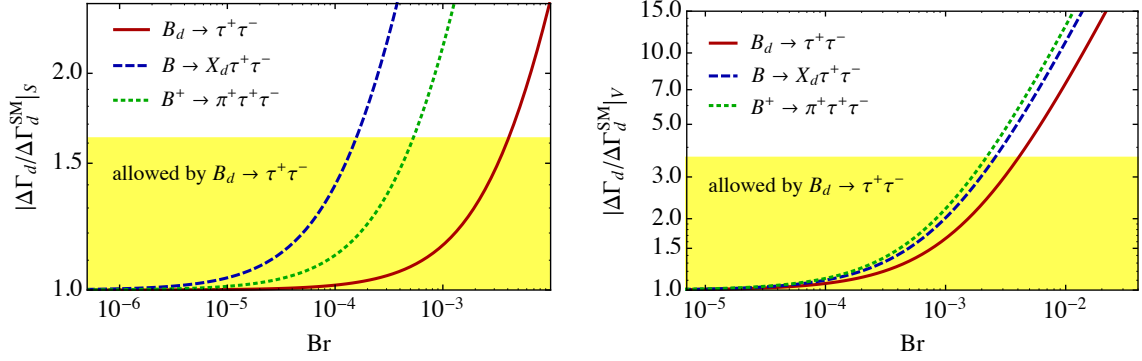
$$\begin{aligned}
|\tilde{\Delta}_d|_{S,AB} &< 1 + (0.41_{-0.08}^{+0.13}) |C_{S,AB}(m_b)|^2, \\
|\tilde{\Delta}_d|_{V,AB} &< 1 + (0.42_{-0.08}^{+0.13}) |C_{V,AB}(m_b)|^2, \\
|\tilde{\Delta}_d|_{T,A} &< 1 + (3.81_{-0.74}^{+1.21}) |C_{T,A}(m_b)|^2,
\end{aligned} \tag{5.15}$$

where the quoted uncertainties are related to the theory error of  $\Delta\Gamma_d^{\text{SM}}$ . The numerical input values of the bag parameters  $B_V$ ,  $B_S$  and  $\tilde{B}_S$  are given in Table 3. Using the strongest bounds from Table 2, i.e.  $|C_{S,AB}(m_b)| \lesssim 1.1$ ,  $|C_{V,AB}(m_b)| \lesssim 2.2$ ,  $|C_{T,L}(m_b)| \lesssim 0.1$  and  $|C_{T,R}(m_b)| \lesssim 0.2$ , results in

$$|\tilde{\Delta}_d|_{S,AB} \lesssim 1.6, \quad |\tilde{\Delta}_d|_{V,AB} \lesssim 3.7, \quad |\tilde{\Delta}_d|_{T,L} \lesssim 1.05, \quad |\tilde{\Delta}_d|_{T,R} \lesssim 1.2. \tag{5.16}$$

These numbers imply that the scalar operators can lead to an enhancement of about 60% over the SM prediction, whereas in the case of vector operators even deviations in excess of 270% are allowed. The possible deviations due to tensor operators can, on the other hand, amount to at most 20%. Such small effects are undetectable given that the hadronic uncertainty in  $\Delta\Gamma_d$  is of similar size.

It is also interesting to study the impact future improved extractions of  $\text{Br}(B_d \rightarrow \tau^+\tau^-)$ ,  $\text{Br}(B \rightarrow X_d\tau^+\tau^-)$  and  $\text{Br}(B^+ \rightarrow \pi^+\tau^+\tau^-)$  will have on the maximal enhancements in  $\Delta\Gamma_d$ . Such a comparison is provided in Figure 5 for the scalar operators (left panel) and the vector operators (right panel). The plots show that for both the scalar and the vector operators and fixed branching ratio the  $B_d \rightarrow \tau^+\tau^-$  decay always provides the most stringent constraint on  $|\Delta\Gamma_d/\Delta\Gamma_d^{\text{SM}}|$ . This implies that in order to restrict possible new-physics effects in  $\Delta\Gamma_d$ , future measurements of the  $B \rightarrow X_d\tau^+\tau^-$  or  $B^+ \rightarrow \pi^+\tau^+\tau^-$  branching ratio have to surpass the present bound (5.5) on  $\text{Br}(B_d \rightarrow \tau^+\tau^-)$ . Numerically, we find



**Figure 5.** 90% CL bounds on possible enhancements of  $\Delta\Gamma_d$  induced by the different  $(\bar{d}b)(\bar{\tau}\tau)$  operators. The left panel shows the effect of scalar operators, while the right panel illustrates the case of vector operators. In both panels the yellow region indicates the maximal enhancements that are consistent with (5.5). The effect of an experimental improvement in the  $B_d \rightarrow \tau^+\tau^-$ ,  $B \rightarrow X_d\tau^+\tau^-$  and  $B^+ \rightarrow \pi^+\tau^+\tau^-$  branching ratios is indicated by the solid red, the dashed blue and the dotted green curves, respectively.

that limits of  $\text{Br}(B^+ \rightarrow \pi^+\tau^+\tau^-) \lesssim 5.3 \cdot 10^{-4}$  and  $\text{Br}(B \rightarrow X_d\tau^+\tau^-) \lesssim 2.6 \cdot 10^{-3}$  would be required in the case of the scalar and vector operators to reach the current sensitivity of the  $B_d \rightarrow \tau^+\tau^-$  branching ratio.

## 6 Conclusion

In this article we have investigated the room for new-physics effects in the decay rate difference  $\Delta\Gamma_d$  of neutral  $B_d$  mesons following an effective field theory approach. Such a study is well-motivated because the current direct experimental bound on  $\Delta\Gamma_d$  still allows for an enhancement of several 100% over the SM prediction and a new measurement at DØ can be interpreted as a solution of the longstanding problem with the dimuon asymmetry, involving an anomalous enhancement of the decay rate difference.

We have presented a detailed comparison between  $\Delta\Gamma_d$  and  $\Delta\Gamma_s$  within the SM, emphasising that while in the former case the relevant CKM factors  $V_{cd}^*V_{cb}$  and  $V_{ud}^*V_{ub}$  both scale as  $\lambda^3$ , in the latter case there is a hierarchy between the individual contributions, since  $V_{cs}^*V_{cb} = \mathcal{O}(\lambda^2)$  and  $V_{us}^*V_{ub} = \mathcal{O}(\lambda^4)$ . In consequence, a modification in  $b \rightarrow c\bar{c}d$  can have a much larger effect in  $\Delta\Gamma_d$ , compared to the effect of a similar modification in  $b \rightarrow c\bar{c}s$  on  $\Delta\Gamma_s$ . Such modifications could for instance arise in new-physics scenarios that predict violations of the CKM unitarity. If these non-standard corrections affect the  $B_d$ -meson and  $B_s$ -meson sectors in a flavour-universal way, then  $\Delta\Gamma_d$  can be enhanced relative to the SM by up to 300%, while in the case of  $\Delta\Gamma_s$  the corresponding shifts can be 50% at most. Our general arguments show that for  $\Delta\Gamma_d$  a large enhancement is a priori not excluded, while in the case of  $\Delta\Gamma_s$  beyond the SM effects cannot dramatically exceed the size of the hadronic uncertainties that plague the decay rate differences.

Our general findings have been corroborated by model-independent studies of non-standard effects associated to dimension-six operators of flavour content  $(\bar{d}p)(\bar{p}'b)$  with

$p, p' = u, c$  as well as  $(\bar{d}b)(\bar{\tau}\tau)$ . Allowing for flavour-dependent and complex Wilson coefficients, we performed detailed analyses of the experimental constraints on each individual coefficient of the current-current operators that arise from hadronic  $B$ -meson decays like  $B \rightarrow \pi\pi, \rho\pi, \rho\rho, D^*\pi$ , the inclusive  $B \rightarrow X_d\gamma$  transition and the dimension-eight contributions to the mixing-induced CP asymmetry in  $B \rightarrow J/\psi K_S$ . In accordance with our general observations, we found that large relative deviations in  $\Delta\Gamma_d$  and  $a_{sl}^d$  of a few 100% can at present not be ruled out phenomenologically, if they are associated to the current-current operator  $(\bar{d}c)(\bar{c}b)$ . We stressed that the derived bounds also apply to new-physics models that feature violations of the unitarity of the CKM matrix, a scenario more restrictive than our own, which allows for non-universal, complex Wilson coefficients.

As a second possibility, we examined new-physics contributions from effective interactions of the form  $(\bar{d}b)(\bar{\tau}\tau)$ . Depending on their Lorentz structure such operators contribute at tree level to the decays  $B_d \rightarrow \tau^+\tau^-$ ,  $B \rightarrow X_d\tau^+\tau^-$  and  $B^+ \rightarrow \pi^+\tau^+\tau^-$ . While for  $B_d \rightarrow \tau^+\tau^-$  there exists a direct experimental bound, the branching ratios of  $B \rightarrow X_d\tau^+\tau^-$  and  $B^+ \rightarrow \pi^+\tau^+\tau^-$  can only be bounded indirectly by using the lifetime difference of  $B_d$  and  $B_s$  mesons. Loop-induced contributions to decays like  $B \rightarrow X_d\gamma$ ,  $B^+ \rightarrow \pi^+\mu^+\mu^-$  and  $B_d \rightarrow \gamma\gamma$  also arise from  $(\bar{d}b)(\bar{\tau}\tau)$  operators, but turn out to be relevant only for tensor interactions. Since the existing experimental constraints are all fairly weak, we found that enhancements of both  $\Delta\Gamma_d$  and  $a_{sl}^d$  by more than 100% are possible also in the case of non-standard  $b \rightarrow d\tau^+\tau^-$  effects, in particular, if these new contributions stem from vector operators. Given that some of the studied  $B_{d,s}$ -meson decays are expected to be better measured or bounded in the near future by LHCb and eventually also Belle II, we also presented plots that illustrate the sensitivity of the individual decay channels to the different Wilson coefficients. These results should prove useful in monitoring the impact that further improved determinations of  $B_d \rightarrow \tau^+\tau^-$ ,  $B \rightarrow X_d\tau^+\tau^-$  and  $B^+ \rightarrow \pi^+\tau^+\tau^-$  have in extracting information on  $\Delta\Gamma_d$ .

We believe that the decay rate difference  $\Delta\Gamma_d$  provides a unique way to probe “exotic” new physics that leads to composite dimension-six operators like  $(\bar{d}c)(\bar{c}b)$  or  $(\bar{d}b)(\bar{\tau}\tau)$ . While explicit new-physics realisations that give rise to such interactions are difficult to construct, based on existing observations the possibility that non-standard effects of this type enhance  $\Delta\Gamma_d$  and in this way explain the observed anomalously large dimuon asymmetry  $A_{CP}$  cannot be excluded. Direct measurements of  $\Delta\Gamma_d$ , more precise data on the individual semi-leptonic CP asymmetries  $a_{sl}^{d,s}$  and the lifetime difference  $\tau_{B_s}/\tau_{B_d}$ , improvements in the experimental determinations of rare and radiative  $b \rightarrow d$  decays, but also a better knowledge of the values of the CKM matrix elements  $V_{cd}$  and  $V_{cs}$  would shed light on this issue. Such measurements (see also [17]) should hence be pursued with vigour.

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## A SM result for $\Delta\Gamma_d$

Within the framework of the HQE, the off-diagonal element  $\Gamma_{12}^d$  of the decay rate matrix can be expressed as a double expansion in the inverse of the heavy bottom-quark mass and in the strong coupling constant

$$\Gamma_{12}^d = \sum_{i=3}^{\infty} \sum_{j=0}^{\infty} \left( \frac{\Lambda_{\text{QCD}}}{m_b} \right)^i \left( \frac{\alpha_s}{4\pi} \right)^j \Gamma_i^{d,(j)}. \quad (\text{A.1})$$

The terms  $\Gamma_3^{d,(j)}$  encode the contributions from dimension-six operators, whose matrix elements are parameterised in terms of the decay constant  $f_{B_d}$  and the bag parameters. The leading perturbative part  $\Gamma_3^{d,(0)}$  is already known since the 80's, while the NLO corrections  $\Gamma_3^{d,(1)}$  were calculated in [66–68]. In the sub-leading HQE corrections  $\Gamma_4^{d,(j)}$ , dimension-seven operators appear. Some of them can be rewritten in terms of dimension-six operators, but others have to be estimated in the vacuum insertion approximation (VIA). A first step in the non-perturbative determination of the dimension-seven operators has been made in [69, 70]. The perturbative contributions  $\Gamma_4^{d,(0)}$  were calculated in [71]. Even the sub-sub-leading corrections  $\Gamma_5^{d,(0)}$  were estimated in [72]. Because of our ignorance concerning the size of the matrix elements of some of the appearing dimension-eight operators we will not use the latter estimates, which in any case result in small corrections only.

Including the corrections mentioned above, we find the following SM prediction for the width difference in the  $B_d$ -meson system

$$\begin{aligned} (\Delta\Gamma_d)_{\text{SM}} &= 0.0029 \text{ ps}^{-1} \left( 1 \pm 0.16_{B_{R_2}} \pm 0.14_{f_{B_d}} \pm 0.07_{\gamma} \pm 0.07_{\mu} \pm 0.05_{\tilde{B}_S} \pm 0.04_{B_{R_0}} \right. \\ &\quad \left. \pm 0.03_{V_{cb}} \pm 0.03_{B_V} \pm 0.01_{m_b} \pm 0.01_{m_c/m_b} \pm 0.01_{|V_{ub}/V_{cb}|} \right) \quad (\text{A.2}) \\ &= (0.0029 \pm 0.0007) \text{ ps}^{-1}. \end{aligned}$$

In order to obtain this result we have used the same numerical input as in [4]. We see that combining the individual sources of uncertainty in quadrature,  $\Delta\Gamma_d$  is known with a precision of around 25% within the SM.

The dominant uncertainty in (A.2) arises from matrix elements of dimension-seven operators denoted by  $R_2$  and  $R_0$  (see [19] for the precise definitions). These contributions gives rise to an uncertainty of 16% and 4%, respectively. One should keep in mind that the corresponding bag parameters have been estimated in [4] quite conservatively by allowing a 50% deviation from the VIA value of 1. Allowing only for modifications of 25%, would reduce the errors corresponding to dimension-seven operators by a factor of 2. While the

Parameter	Value	Unit	Ref.	Parameter	Value	Unit	Ref.
<b>Masses and couplings</b>							
$m_\mu$	0.105	GeV	[73]	$G_F$	$1.16638 \cdot 10^{-5}$	$\text{GeV}^{-2}$	[73]
$m_\tau$	1.777	GeV	[73]	$\alpha_s(M_Z)$	0.1184		[73]
$M_Z$	91.1876	GeV	[73]	$\alpha(M_Z)$	1/127.944		[73]
$M_W$	80.385	GeV	[73]	$\alpha(m_b)$	1/132		
<b>CKM</b>							
$\lambda$	$0.22457^{+0.00186}_{-0.00014}$		[27]	$\gamma$	$70 \pm 10$	$^\circ$	[27]
$\rho$	$0.1289^{+0.0176}_{-0.0094}$		[27]	$\eta$	$0.348 \pm 0.012$		[27]
$ V_{td}^* V_{tb} $	0.00874			$ V_{cb} $	0.0416		
$ V_{ud} $	0.9745		[27]				
<b>Quark masses</b>							
$m_d$	4.8	MeV		$\overline{m}_c(\overline{m}_c)$	$1.275 \pm 0.025$	GeV	[73]
$m_b^{\text{pole}}$	4.8	GeV		$\overline{m}_b(\overline{m}_b)$	4.2	GeV	
$m_t^{\text{pole}}$	$173.1 \pm 0.9$	GeV	[73]	$\overline{m}_t(\overline{m}_t)$	163.5	GeV	
<b>B-meson and light meson properties</b>							
$M_{B_u}$	5279.26	MeV	[73]	$\tau_{B_u}$	1.641	ps	[73]
$M_{B_d}$	5279.58	MeV	[73]	$\tau_{B_d}$	1.519	ps	[73]
$M_{B_s}$	5366.77	MeV	[73]	$\tau_{B_s}$	1.463	ps	[73]
$f_{B_{u,d}}$	$190.5 \pm 4.2$	MeV	[74]	$f_{B_s}$	$227.7 \pm 4.5$	MeV	[74]
$f_\pi$	130.4	MeV	[73]	$\Lambda_B$	400	MeV	[41]
$B_V$	$0.84 \pm 0.07$	GeV	[74]	$B_S$	$1.36 \pm 0.14$	GeV	[80]
$\tilde{B}_S$	$1.44 \pm 0.16$	GeV	[81]				

**Table 3.** Collection of our input parameters for the numerical evaluation.

bag parameters of some of the dimension-seven operators have been estimated in [69, 70] using QCD sum rules, a lattice calculation of  $B_{R_2}$  and  $B_{R_0}$  is unfortunately not available at present. Any progress in this direction would be of utmost importance to reduce the total uncertainty in  $(\Delta\Gamma_d)_{\text{SM}}$ .

The second largest uncertainty in (A.2) arises from the matrix elements of dimension-six operators. An error of 14% comes from the decay constant  $f_{B_d}$  and an uncertainty of 5% (3%) stems from the bag parameter  $\tilde{B}_S$  ( $B_V$ ). Several lattice groups are working on the determination of these hadronic parameters, so that these errors are expected to shrink.

Number three in the error budget of  $(\Delta\Gamma_d)_{\text{SM}}$  are the CKM uncertainties. The error due to  $\gamma$  ( $V_{cb}$ ) amounts to 7% (3%) at present, but is expected to improve in the future. The renormalisation scale ( $\mu$ ) dependence gives rise to an uncertainty of about 7%. To reduce this error a NNLO order calculation would be necessary. The remaining parametric uncertainties in (A.2) are negligible at the current stage.

## B Numerical input

In Table 3 we collect the numerical values of various parameters used as input throughout the work. The decay constants  $f_{B_q}$  are the most recent averages of the FLAG compilation [74] that incorporate the  $N_f = 2 + 1$  results of [75–77]. More recent determinations with  $N_f = 2 + 1 + 1$  [78] and  $N_f = 2$  [79] are consistent with these averages.

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